Chapter 4 **Quadratic Relations**

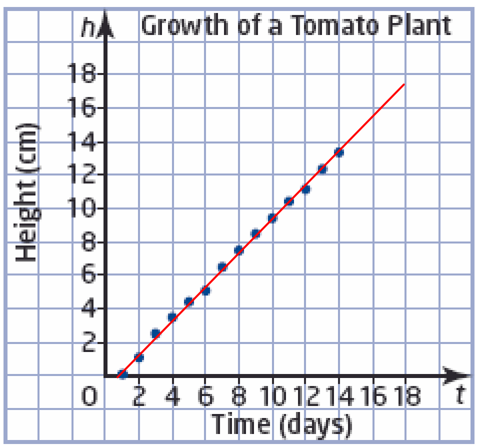
Chapter 4 Get Ready

Chapter 4 Get Ready **Question 1** **Page 162**

a) The independent variable is time. The dependent variable is the height.

Time (days)	1	2	3	4	5	6	7
Height (cm)	0.4	1.2	2.5	3.4	4.3	5.2	6.5
Time (days)	8	9	10	11	12	13	14
Height (cm)	7.5	8.5	9.3	10.3	11.2	12.4	13.4

b)



c) The relationship between the variables appears linear. The points lie close to a straight line.

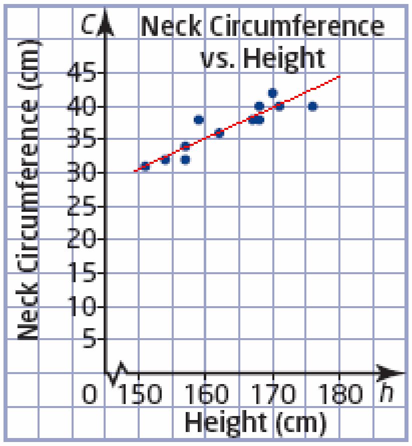
d) Extrapolate the graph. The height of the plant after 17 days is about 16.4 cm.

Chapter 4 Get Ready **Question 2** **Page 162**

a) The independent variable is the height. The dependent variable is the neck circumference.

Height (cm)	157	168	162	151	157	170
Neck Circumference (cm)	32	40	36	31	34	42
Height (cm)	167	159	168	171	176	154
Neck Circumference (cm)	38	38	38	40	40	32

b)

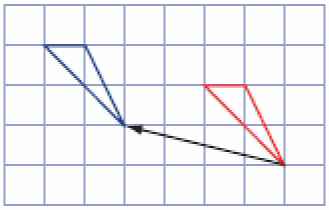


c) The relationship between the variables appears linear. The points lie close to a straight line.

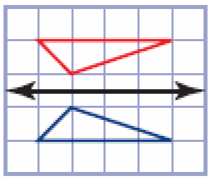
d) Extrapolate the graph. The neck circumference for a 180-cm tall student is about 44 cm.

Chapter 4 Get Ready **Question 3** **Page 163**

The red figure is translated 4 units left and 1 unit up to the blue figure.



Chapter 4 Get Ready **Question 4** **Page 163**



Chapter 4 Get Ready **Question 5** **Page 163**

a) $2^3 \times 2^4 = 2^{3+4}$
$= 2^7$

b) $(-1)^2 \times (-1)^5 = (-1)^{2+5}$
$= (-1)^7$

c) $\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{2+3}$
$= \left(\frac{1}{2}\right)^5$

d) $5^8 \div 5^3 = 5^{8-3}$
$= 5^5$

e) $(-3)^7 \div (-3)^4 = (-3)^{7-4}$
$= (-3)^3$

f) $(4^2)^5 = 4^{2\times5}$
$= 4^{10}$

Chapter 4 Get Ready **Question 6** **Page 163**

a) $2^3 \times 2^4 \div 2^5 = 2^{3+4-5}$
$= 2^2$

b) $(-3)^9 \div (-3)^5 \times (-3)^2 = (-3)^{9-5+2}$
$= (-3)^6$

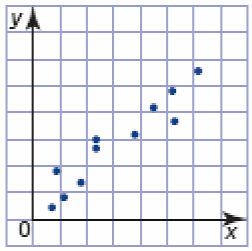
c) $(5^2)^4 \div 5^3 = 5^{2\times4-3}$
$= 5^5$

d) $4^7 \times 4^3 \div (4^2)^4 = 4^{7+3-2\times4}$
$= 4^2$

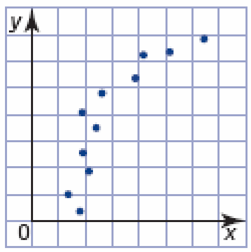
Chapter 4 Section 1: Investigate Non-Linear Relations

Chapter 4 Section 1 Question 1 Page 166

a)



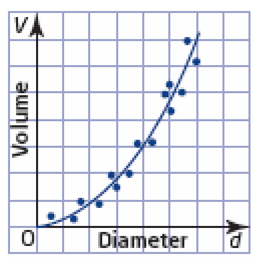
b)



The scatter plot in part b) could be modelled using a curve instead of a line of best fit. The points do not lie along a line.

Chapter 4 Section 1 Question 2 Page 166

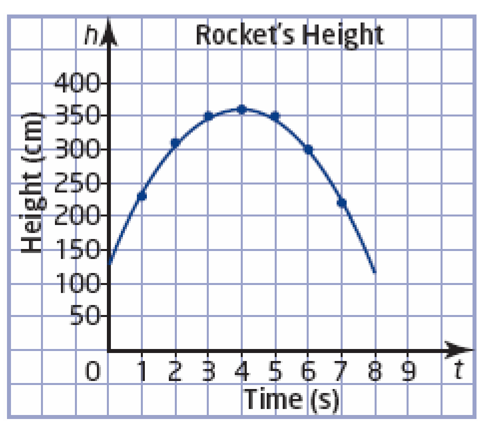
This relation is non-linear. The points lie on a curve.



Chapter 4 Section 1 Question 3 Page 167

a)

Time (s)	1	2	3	4	5	6	7
Height (m)	230	310	350	360	350	300	220



b) The relation is non-linear. The points lie along a curve.

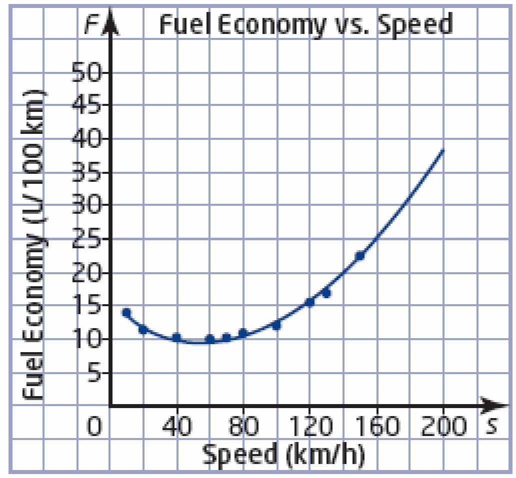
c) The curve of best fit is shown.

d) Answers may vary. For example: The height of the rocket after 8 s is about 111 m.

Chapter 4 Section 1 **Question 4** **Page 167**

a)

Speed (km/h)	Fuel Economy (L/100 km)
10	14.26
20	12.85
40	10.65
60	10.10
70	10.24
80	10.84
100	12.14
120	15.64
130	16.88
150	22.50



b) The relation is non-linear. The points lie on a curve.

c) The curve of best fit is shown.

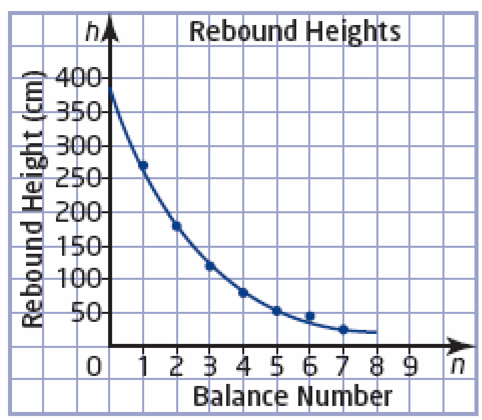
d) Answers may vary. For example: The fuel economy at 200 km/h is about 40 L/100 km.

e) Answers may vary. For example: The graph for a car with better fuel economy would be translated down compared to the graph in part a).

Chapter 4 Section 1 **Question 5** **Page 167**

a)

Bounce Number	1	2	3	4	5	6	7
Rebound Height (cm)	270	180	120	80	53	45	25



b) The relation is non-linear. The points lie on a curve.

c) The curve of best fit is shown.

d) If the ball were bouncier, the rebound heights would not decrease as quickly as shown in this graph.

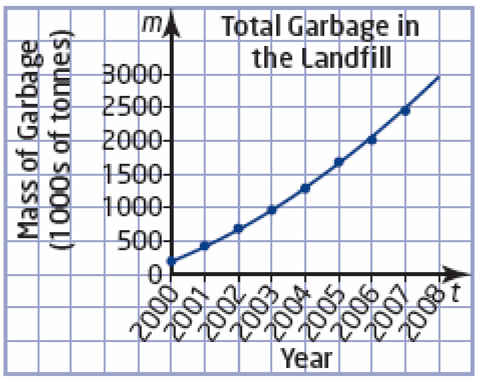
Chapter 4 Section 1 Question 6 Page 167

a)

Year	Garbage Added (1000s of tonnes)
2000	200
2001	230
2002	258
2003	287
2004	317
2005	347
2006	376
2007	406

Year	Total Garbage (1000s of tonnes)
2000	200
2001	430
2002	688
2003	975
2004	1292
2005	1639
2006	2015
2007	2421

b)



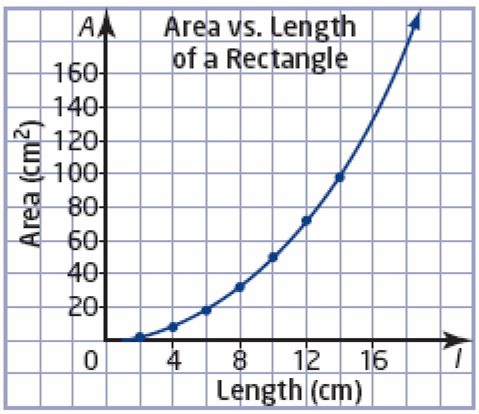
c) Answers will vary. For example: If the growth continues at its current rate, the city will run out of landfill space.

Chapter 4 Section 1 **Question 7** **Page 167**

a)

Length (cm)	Area (cm^2)
2	2
4	8
6	18
8	32
10	50
12	72
14	98
16	128

b)



c) The curve of best fit is shown on the graph.

d) Answers may vary. For example:

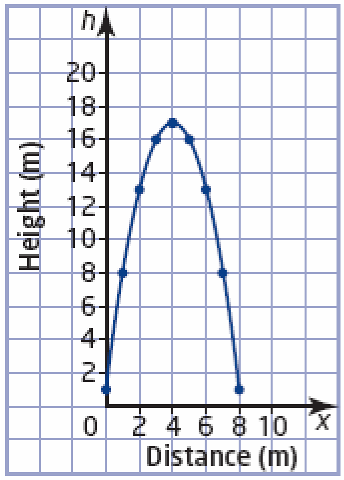
This relation is non-linear because length is a linear measurement but area is a square measurement.

Chapter 4 Section 2 Quadratic Relations

Chapter 4 Section 2 Question 1 Page 172

a)

x	h
0	1
1	8
2	13
3	16
4	17
5	16
6	13
7	8
8	1



b) The flight path of the ball is parabolic. The axis of symmetry is $x = 4$, and the vertex is (4, 17).

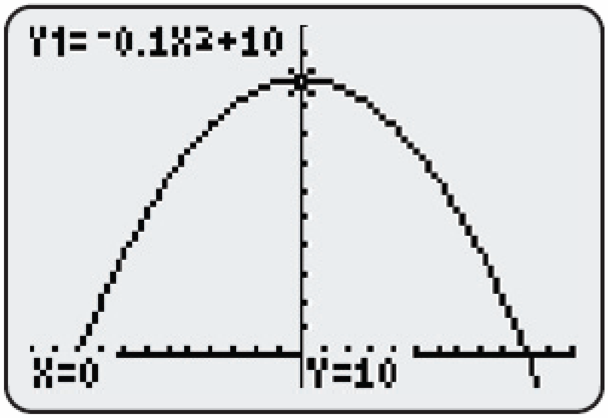
c) The maximum height reached is 17 m.

d) Construct a table of values for $h = -x^2 + 8x + 1$. The same values as those in the previous table are generated. The equation can be used to model the flight path of the ball.

x	h
0	1
1	8
2	13
3	16
4	17
5	16
6	13
7	8
8	1

Chapter 4 Section 2 **Question 2 Page 172**

a)



b) The shape of the arch is parabolic.

c) The arch is 10 m tall and 20 m wide.

Chapter 4 Section 2 Question 3 Page 172

a)

x	y	First Differences
0	4	
1	5	1
2	6	1
3	7	1
4	8	1

First differences are constant. The relation is linear.

b)

x	y	First Differences	Second Differences
0	3		
1	4	1	
2	7	3	2
3	12	5	2
4	19	7	2

Second differences are constant. The relation is quadratic.

c)

x	y	First Differences	Second Differences
1	0		
3	1	1	
5	8	7	6
7	27	19	12
9	64	37	18

Neither first nor second differences are constant. The relation is neither linear nor quadratic.

d)

x	y	First Differences	Second Differences
–2	6		
1	0	–6	
4	12	12	18
7	42	30	18
10	90	48	18

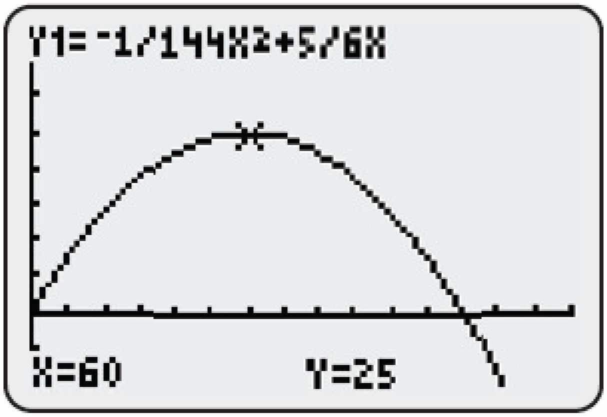
Second differences are constant. The relation is quadratic.

Chapter 4 Section 2 Question 4 Page 172

Answers will vary.

Chapter 4 Section 2 Question 5 Page 172

a)



b) Read the value from your graph or table of values or use the TRACE and ZOOM features on the graphing calculator. When x is 12, y is 9.

The height of the bridge 12 m horizontally from one end is 9 m.

c) Read the value from your graph or table of values or use the TRACE and ZOOM features on the graphing calculator. The curve crosses the x-axis at 0 and 120.

The bridge is 120 m wide at its base.

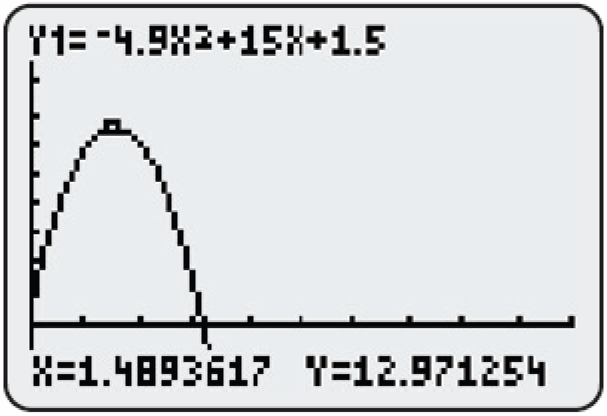
d) Read the value from your graph or table of values or use the TRACE and ZOOM features on the graphing calculator. The maximum value of y is 25, when x is 60.

The maximum height of the bridge is 25 m at a horizontal distance of 60 m.

e) The parabola is symmetrical about a vertical line $x = 60$.

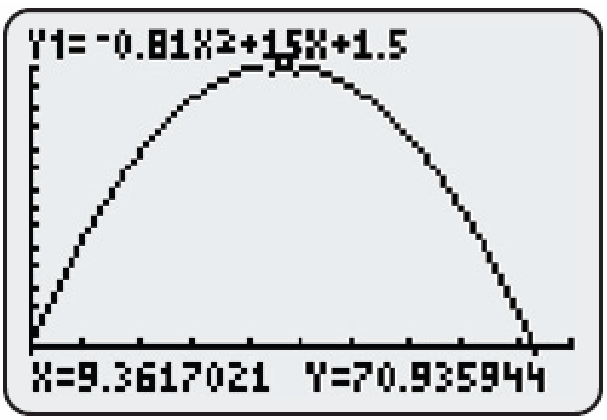
Chapter 4 Section 2 **Question 6 Page 173**

a)



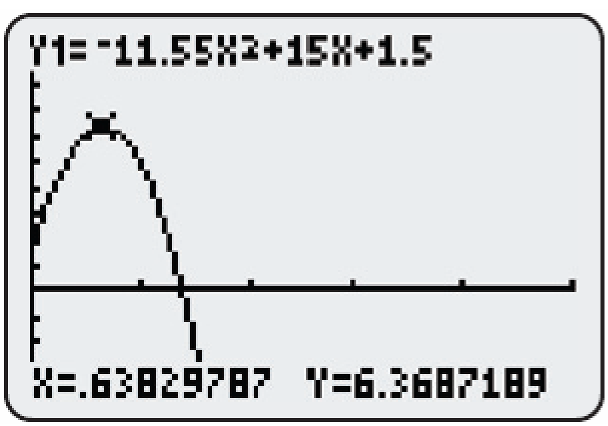
b) The relation between time and height is quadratic.

c)



There is a quadratic relation.

d)



There is a quadratic relation.

e) All three graphs show a quadratic relation between time and height. The parabolas have different axes of symmetry and different vertices. Since the ball falls to the ground faster on Jupiter than on Earth and the Moon, Jupiter has a stronger force of gravity than Earth, and Earth has a stronger force of gravity than the Moon.

Chapter 4 Section 2 Question 7 Page 173

Year	Total Garbage (1000s of tonnes)	First Differences	Second Differences
2000	200		
2001	430	230	
2002	688	258	28
2003	975	287	29
2004	1292	317	30
2005	1639	347	30
2006	2015	376	29
2007	2421	406	30

The second differences are close to being constant. The relation is closely modelled using a quadratic relation.

Chapter 4 Section 2 Question 8 Page 173

x	y	First Differences	Second Differences
−1.5	1.0		
−1.0	1.5	0.5	−0.25
−0.5	1.75	0.25	0.25
0.0	2.25	0.50	−0.35
0.5	2.4	0.15	−0.30
1.0	2.25	−0.15	0.10
1.5	2.2	−0.05	−0.15
2.0	2.0	−0.20	−0.05
2.5	1.75	−0.25	−1.00
3.0	0.75	−1.25	

The arch does not closely resemble a parabola. The second differences are not constant.

Chapter 4 Section 2 Question 9 Page 173

Solutions for Achievement Checks are shown in the Teacher's Resource.

Chapter 4 Section 2 Question 10 Page 173

$$\begin{aligned} r &= 2d^2 \\ &= 2(0.3)^2 \\ &= 0.18 \end{aligned}$$

The flow rate is 0.18 L/s.

It would take $\frac{200}{0.18}$, or about 1111 s (18 min 31 s) to fill a 200-L barrel.

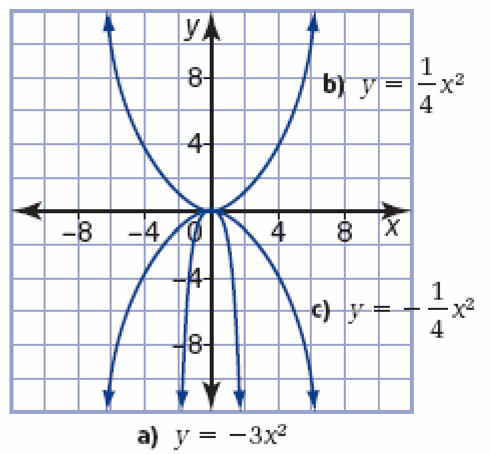
Chapter 4 Section 2 Question 11 Page 173

The sum of the first n natural numbers is given by the relation $y = \frac{n(n+1)}{2}$.

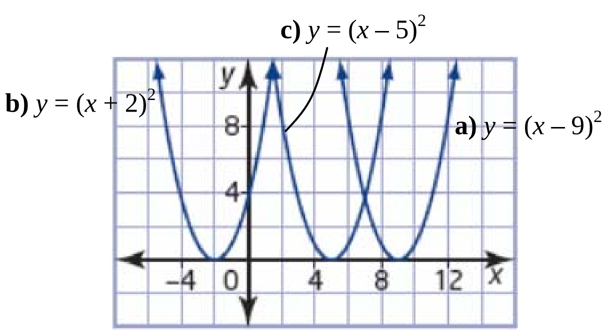
n	Sum	*y*
1	1	1
2	1 + 2 = 3	3
3	1 + 2 + 3 = 6	6
4	1 + 2 + 3 + 4 = 10	10
5	1 + 2 + 3 + 4 + 5 = 15	15
6	1 + 2 + 3 + 4 + 5 + 6 = 21	21

Chapter 4 Section 3 **Investigate Transformations of Quadratics**

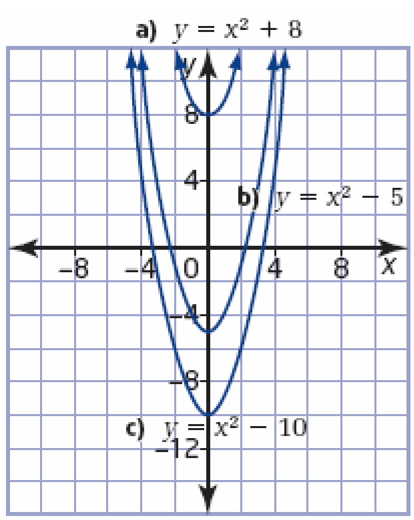
Chapter 4 Section 3 **Question 1** **Page 178**



Chapter 4 Section 3 **Question 2** **Page 178**

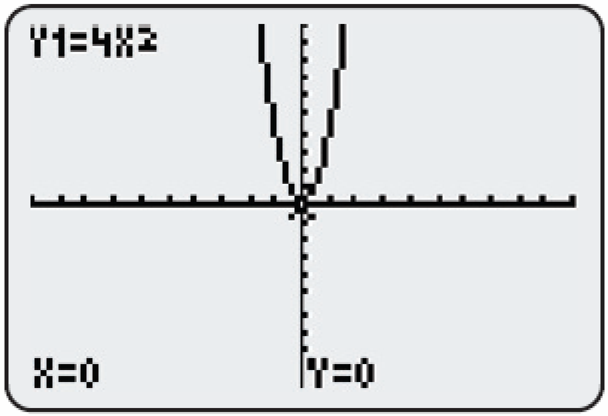


Chapter 4 Section 3 **Question 3** **Page 178**



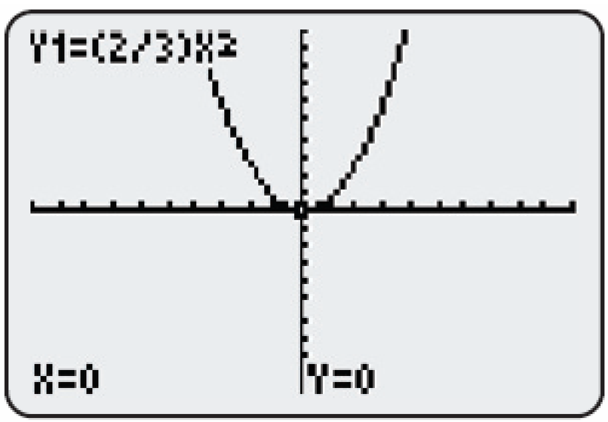
Chapter 4 Section 3 **Question 4 Page 178**

a)



Three points are (–1, 4), (0, 0), and (1, 4).
The parabola is vertically stretched by a factor of 4.

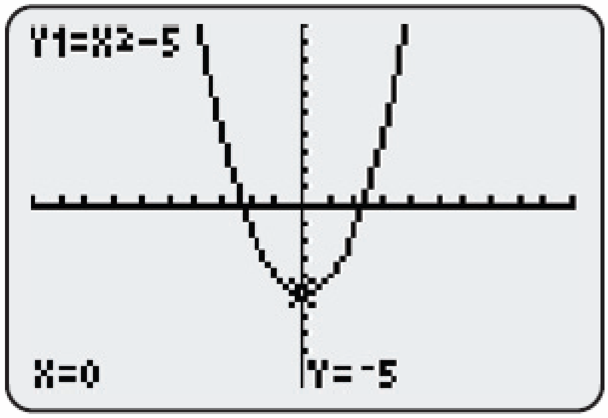
b)



Three points are (–3, 6), (0, 0), and (3, 6).

The parabola is vertically compressed by a factor of $\frac{2}{3}$.

c)

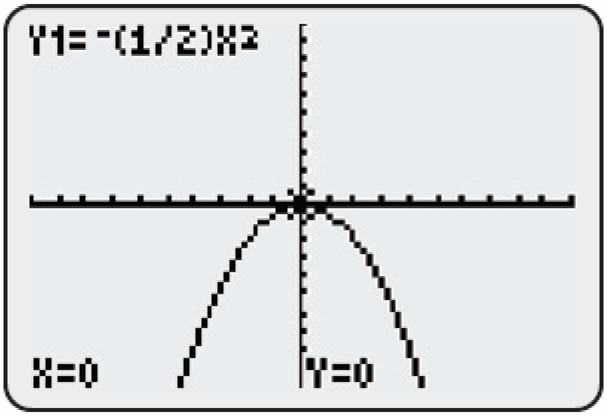


Three points are (–1, –4), (0, –5), and (1, –4).
The parabola is translated 5 units downward.

d)

Three points are (7, 1), (8, 0), and (9, 1).
The parabola is translated 8 units to the right.

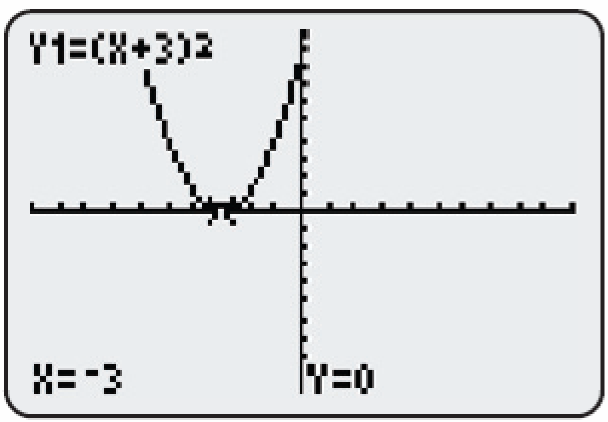
e)



Three points are (–2, –2), (0, 0), and (2, –2)

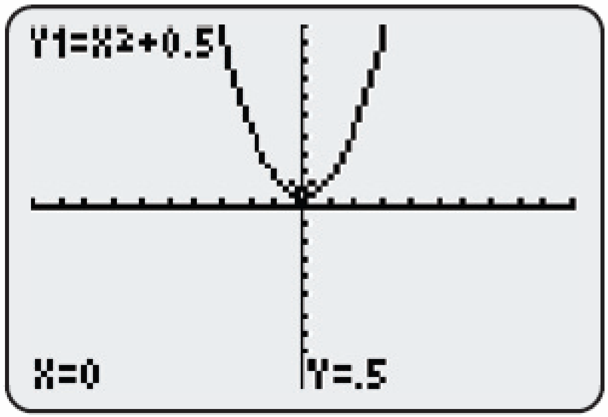
The parabola is vertically compressed by a factor of $\frac{1}{2}$ and reflected in the *x*-axis.

f)



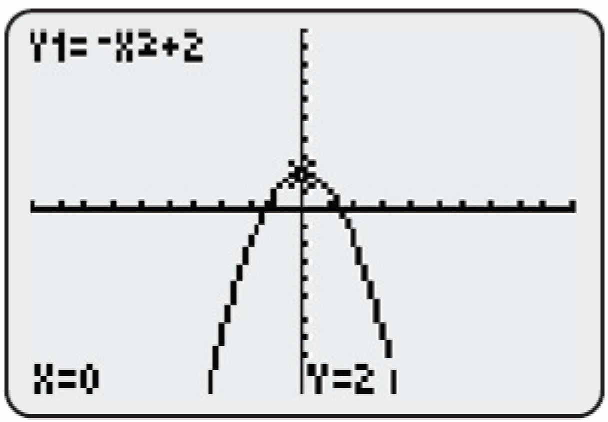
Three points are (–4, 1), (–3, 0), and (–2, 1).
The parabola is translated 3 units to the left.

g)



Three points are (–1, 1.5), (0, 0.5), and (1, 1.5).
The parabola has been translated 0.5 units upward.

h)



Three points are (–1, 1), 0, 2), and (1, 1).
The parabola is reflected in the *x*-axis, and translated 2 units upward.

Chapter 4 Section 3 Question 5 Page 178

a)

x	$y = x^2$	$y = 2x^2$	$y = x^2 + 1$	$y = (x - 3)^2$
−3	9	18	10	36
−2	4	8	5	25
−1	1	2	2	16
0	0	0	1	9
1	1	2	2	4
2	4	8	5	1
3	9	18	10	0

b) The y-values for $y = 2x^2$ are all twice the y-values for $y = x^2$.

c) The y-values for $y = x^2 + 1$ are all one more than the y-values for $y = x^2$.

d) The y-values for $y = (x - 3)^2$ are the same as the y-values for $y = x^2$ for x-values that are 3 greater.

Chapter 4 Section 3 Question 6 Page 178

a) $y = x^2 + 6$

b) $y = x^2 - 4$

Chapter 4 Section 3 Question 7 Page 178

a) $y = (x + 7)^2$

b) $y = (x - 5)^2$

c) $y = (x + 8)^2$

d) $y = (x - 3)^2$

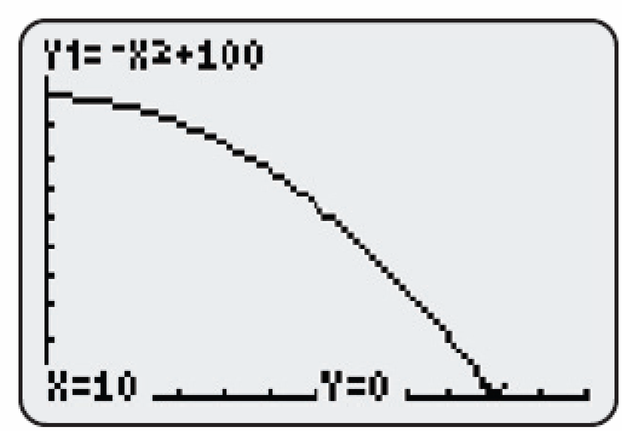
Chapter 4 Section 3 Question 8 Page 179

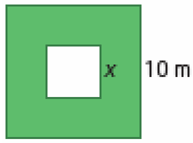
a) $y = 8x^2$

b) $y = \frac{1}{5}x^2$

Chapter 4 Section 3 **Question 9** **Page 179**

a)





b) For the *A*-intercept let *x* = 0.

$A = -x^2 + 100$

$A = -0^2 + 100$

$= 100$

For the *x*-intercept, let *A* = 0.

$A = -x^2 + 100$

$0 = -x^2 + 100$

$x^2 = 100$

$x = 10$

The *A*-intercept (vertical intercept) is 100. This represents the area of the grass if there is no square patio in the centre of the grass.

The *x*-intercept is 10. This represents the side length of the patio, in metres, if the patio completely covers the grass in the backyard.

c) If the side length is 12 m, the equation becomes $A = -x^2 + 144$.

d) For the original equation, $0 \le x \le 10$. For the second equation, $0 \le x \le 12$.

Chapter 4 Section 3 **Question 10** **Page 179**

a) $l = 0.04s^2$

$= 0.04(50)^2$

$= 100$

$l = 0.04s^2$

$= 0.04(100)^2$

$= 400$

The skid mark for a car travelling at 50 km/h is 100 m. The skid mark for a car travelling at 100 km/h is 400 m.

b) When the speed of the car doubles, the length of the skid mark quadruples.

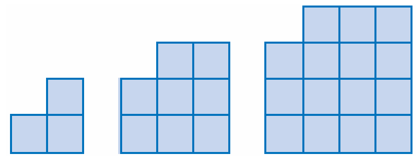
c) *s* must be greater than 0.

d) Answers may vary. For example: If the pavement were wet, the skid marks would be longer. The equation would have a coefficient greater than 0.04.

Chapter 4 Section 3 Question 11 Page 179

a)

l	A	First Differences	Second Differences
2	3		
		5	
3	8		2
		7	
4	15		2
		9	
5	24		2
		11	
6	35		



The second differences are constant. The relation is quadratic.

b) $A = l^2 - 1$

c) The transformation is a translation 1 unit downward.

Chapter 4 Section 3 Question 12 Page 179

a) Answers may vary. For example: According to the order of operations, multiplying by a or adding k is done after squaring the x-value. The transformation applies directly to the parabola $y = x^2$. Because the value of h must be added or subtracted before squaring, the shift is opposite to the sign in the bracket, and must be the opposite movement to get back to the original y-value for the graph of $y = x^2$.

b) The graph of $y = (2x)^2$ is the graph of $y = x^2$ stretched vertically by a factor of 4.

Chapter 4 Section 3 Question 13 Page 179

Substitute the coordinates of the each point to obtain two equations that can be solved for a and k.

For (–1, 3):

$$y = ax^2 + k$$
$$3 = a(-1)^2 + k$$
$$3 = a + k \quad ①$$

For (3, –13):

$$y = ax^2 + k$$
$$-13 = a(3)^2 + k$$
$$-13 = 9a + k \quad ②$$

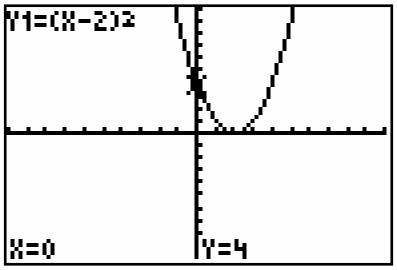
$$3 = a + k \quad ①$$
$$-13 = 9a + k \quad ②$$
$$-16 = 8a \quad ① - ②$$
$$-2 = a$$

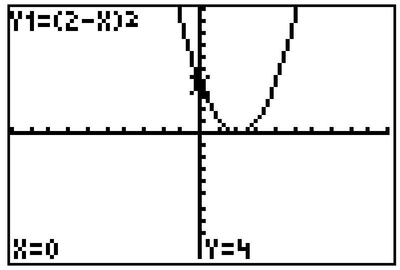
Substitute $a = -2$ into equation ①.

$$3 = a + k$$
$$3 = -2 + k$$
$$5 = k$$

$a = -2$ and $k = 5$.

Chapter 4 Section 3 Question 14 Page 179





The graphs of $y = (x - 2)^2$ and $y = (2 - x)^2$ are identical.

Chapter 4 Section 3 Question 15 Page 179

a) Answers will vary. For example: The graphs are both parabolic.

The graph of $y = (x - 2)^2 + 5$ opens upward and the graph of $x = (y - 2)^2 + 5$ opens to the right.

The vertices are (2, 5) and (5, 2), respectively.

The equations of the axes of symmetry are $x = 2$ and $y = 2$, respectively.

The x and y variables have switched in the equations.

b)

$$\begin{aligned} x &= (y-2)^2 + 5 \\ x - 5 &= (y-2)^2 \\ \pm\sqrt{x-5} &= y - 2 \\ 2 \pm \sqrt{x-5} &= y \end{aligned}$$

Chapter 4 Section 4 **Graph $y = a(x - h)^2 + k$**

Chapter 4 Section 4 **Question 1** **Page 185**

a)

Property	$y = (x - 4)^2$
Vertex	(4, 0)
Axis of symmetry	$x = 4$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 0$

b)

Property	$y = (x - 2)^2 - 4$
Vertex	(2, –4)
Axis of symmetry	$x = 2$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -4$

c)

Property	$y = (x + 3)^2 - 2$
Vertex	(–3, –2)
Axis of symmetry	$x = -3$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -2$

d)

Property	$y = \frac{1}{2}(x + 1)^2 + 5$
Vertex	(–1, 5)
Axis of symmetry	$x = -1$
Stretch or compression factor relative to $y = x^2$	$\frac{1}{2}$
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 5$

e)

Property	$y = (x - 7)^2 - 3$
Vertex	(7, −3)
Axis of symmetry	$x = 7$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -3$

f)

Property	$y = -(x - 1)^2 + 7$
Vertex	(1, 7)
Axis of symmetry	$x = 1$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq 7$

g)

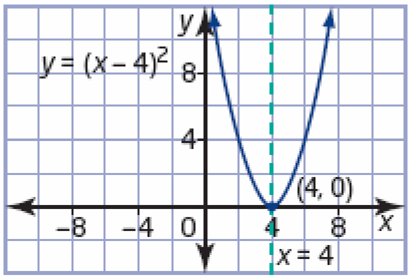
Property	$y = 2(x - 4)^2 - 5$
Vertex	(4, −5)
Axis of symmetry	$x = 4$
Stretch or compression factor relative to $y = x^2$	2
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -5$

h)

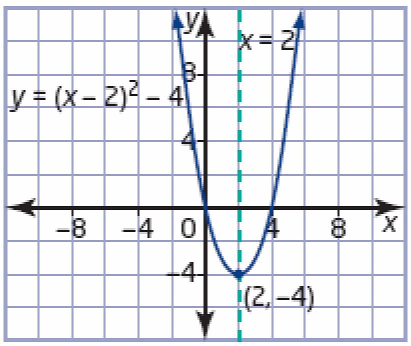
Property	$y = -3(x + 4)^2 - 2$
Vertex	(−4, −2)
Axis of symmetry	$x = -4$
Stretch or compression factor relative to $y = x^2$	3
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq -2$

Chapter 4 Section 4 Question 2 Page 185

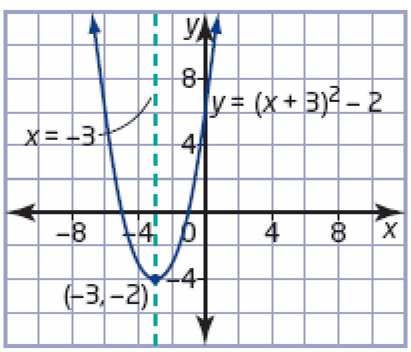
a)



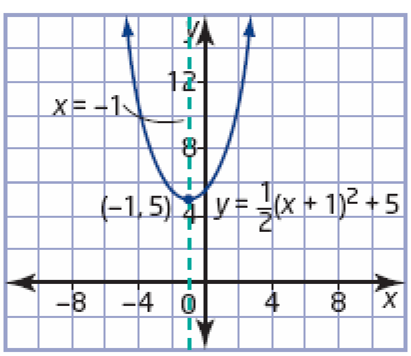
b)



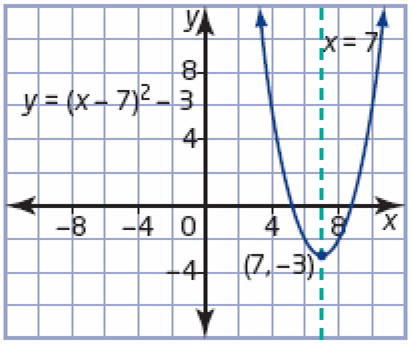
c)



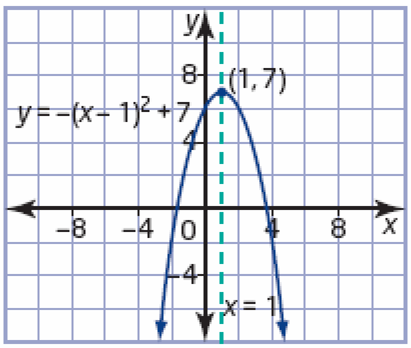
d)



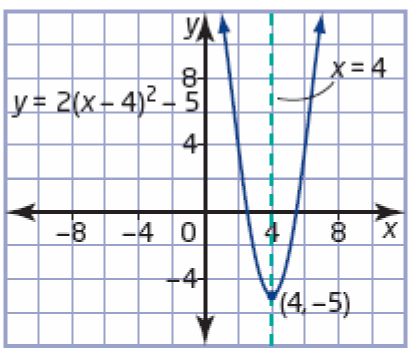
e)


y
x = 7
8
y = (x – 7)^2 – 3
4
–8
–4
0
4
8
x
–4
(7, –3)

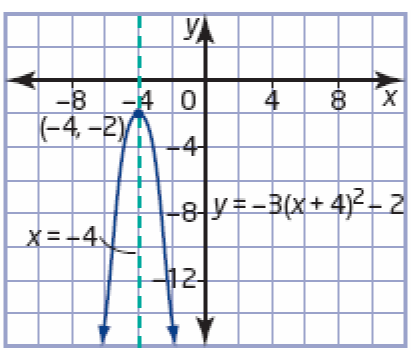
f)


y
8
(1, 7)
y = –(x – 1)^2 + 7
4
–8
–4
0
4
8
x
–4
x = 1

g)


y
x = 4
8
y = 2(x – 4)^2 – 5
4
–8
–4
0
4
8
x
–4
(4, –5)

h)


y
–8
–4
0
4
8
x
(–4, –2)
–4
–8
y = –3(x + 4)^2 – 2
x = –4
–12

Chapter 4 Section 4 Question 3 Page 185

An equation for the parabola is $y = (x - 2)^2 + 3$.

Chapter 4 Section 4 Question 4 Page 185

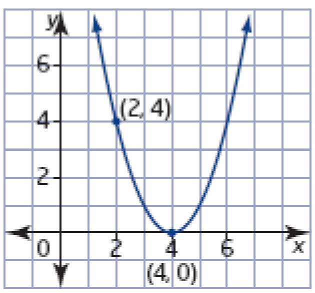
An equation for the parabola is $y = -2(x + 3)^2$.

Chapter 4 Section 4 Question 5 Page 185

An equation for the parabola is $y = 0.3(x - 4)^2 - 1$.

Chapter 4 Section 4 Question 6 Page 185

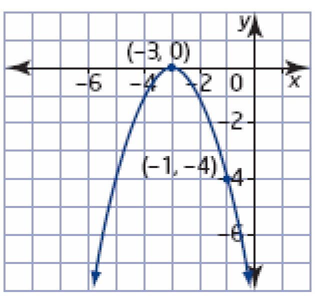
a)



The vertex has been shifted 4 units to the right. There is no vertical stretch or compression.

An equation for the parabola is $y = (x - 4)^2$.

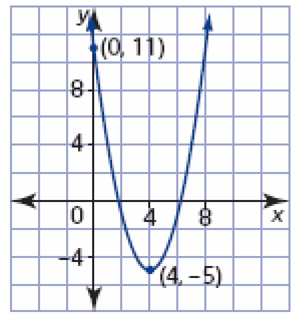
b)



The parabola has been reflected in the *x*-axis. The vertex has been shifted 3 units to the left. There is no vertical stretch or compression.

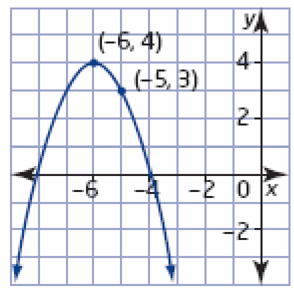
An equation for the parabola is $y = -(x + 3)^2$.

Chapter 4 Section 4 Question 7 Page 186



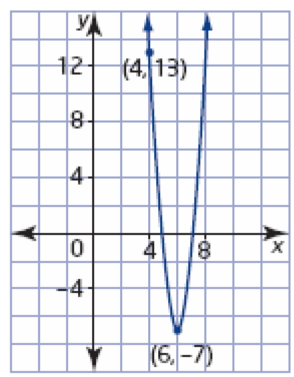
a) The vertex has been shifted 4 units to the right and 5 units down. There is no vertical stretch or compression.

An equation for the parabola is $y=(x-4)^2-5$.



b) The parabola has been reflected in the *x*-axis. The vertex has been shifted 6 units to the left and 4 units up. There is no vertical stretch or compression.

An equation for the parabola is $y=-(x+6)^2+4$.



c) The vertex has been shifted 6 units to the right and 7 units down. There is a vertical stretch.
Substitute *h* = 6 and *k* = –7. Then, substitute *x* = 4 and *y* = 13 and solve for *a*.

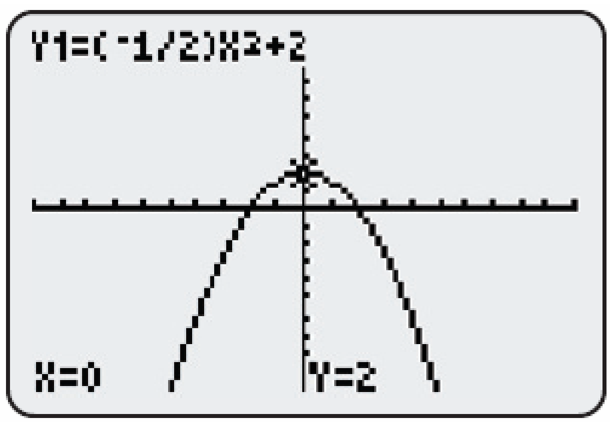
$$y=a(x-h)^2+k$$
$$y=a(x-6)^2+(-7)$$
$$13=a(4-6)^2-7$$
$$13=4a-7$$
$$20=4a$$
$$5=a$$

An equation for the parabola is $y=5(x-6)^2-7$.

Chapter 4 Section 4 Question 8 Page 186

The equation for the parabola is $y = 3(x+2)^2 - 1$.

Chapter 4 Section 4 Question 9 Page 186



The equation for the parabola is $y = -\frac{1}{2}(x)^2 + 2$.

Chapter 4 Section 4 Question 10 Page 186

a) Substitute $h = 1$ and $k = 4$. Then, substitute $x = 3$ and $y = 8$ and solve for a.

$$y = a(x-h)^2 + k$$
$$y = a(x-1)^2 + 4$$
$$8 = a(3-1)^2 + 4$$
$$8 = 4a + 4$$
$$4 = 4a$$
$$1 = a$$

An equation for the parabola is $y = (x-1)^2 + 4$.

b) Substitute $h = –2$ and $k = 5$. Then, substitute $x = 0$ and $y = 1$ and solve for a.

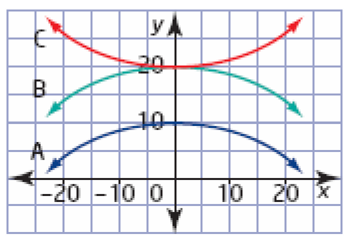
$$y = a(x-h)^2 + k$$
$$y = a(x-(-2))^2 + 5$$
$$1 = a(0+2)^2 + 5$$
$$1 = 4a + 5$$
$$-4 = 4a$$
$$-1 = a$$

An equation for the parabola is $y = -(x+2)^2 + 5$.

Chapter 4 Section 4 Question 11 Page 186

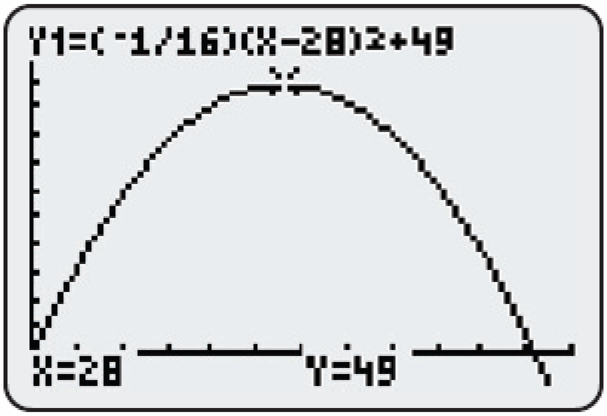
The equation for the parabolic arch is $y = -\frac{1}{45}x^2 + 20$.

The parabola opens downward. The y-intercept is 20. This matches graph B.



Chapter 4 Section 4 **Question 12** **Page 186**

a)



b) The equation is $h = -\frac{1}{16}(d-28)^2 + 49$.

The maximum height of the soccer ball is 49 m.

c) The horizontal distance of the soccer ball is 28 m when it reaches its maximum height.

d)
$$\begin{aligned} h &= -\frac{1}{16}(d-28)^2 + 49 \\ &= -\frac{1}{16}(20-28)^2 + 49 \\ &= -\frac{1}{16}(8)^2 + 49 \\ &= 45 \end{aligned}$$

The height of the soccer ball at a horizontal distance of 20 m is 45 m.

e) In part d) the horizontal distance of 20 m occurs 8 m before the maximum point. Due to the symmetric property of a parabola, the soccer ball is at the same height 8 m after the maximum point, or at a horizontal distance of 36 m.

Chapter 4 Section 4 Question 13 Page 187

a) The batting height represents the point (0, 1). The maximum height of 33 m at a horizontal distance of 4 m represents the vertex, (4, 33). Substitute h = 4 and k = 33. Then, substitute x = 0 and y = 1 and solve for a.

$$y = a(x-h)^2 + k$$
$$y = a(x-4)^2 + 33$$
$$1 = a(0-4)^2 + 33$$
$$-32 = 16a$$
$$-2 = a$$

An equation to model the path of the baseball is $y = -2(x-4)^2 + 33$, or using variables h for height and d for the horizontal distance, $h = -2(d-4)^2 + 33$.

b) $h = -2(d-4)^2 + 33$

$$= -2(6-4)^2 + 33$$
$$= -2(2)^2 + 33$$
$$= 25$$

The height of the baseball at a horizontal distance of 6 m is 25 m.

c) In part b) the horizontal distance of 6 m occurs 2 m after the maximum point. Due to the symmetric property of a parabola, the baseball is at the same height 2 m before the maximum point, or at a horizontal distance of 2 m.

Chapter 4 Section 4 Question 14 Page 187

a) The equation that models the flight path of the firework is $h = -5(t-5)^2 + 127$.

The maximum height of 127 m is reached at a time of 5 s.

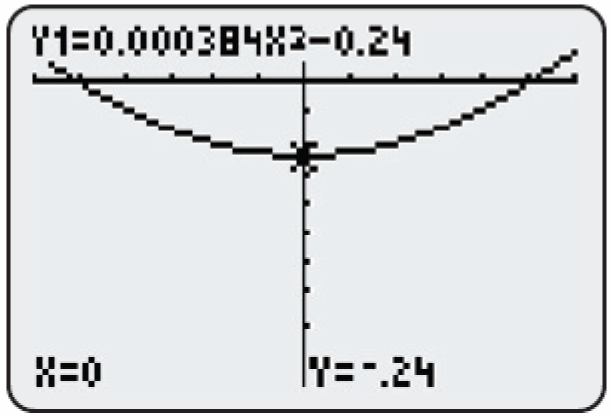
b) When the firework was fired, t = 0.

$$h = -5(t-5)^2 + 127$$
$$= -5(0-5)^2 + 127$$
$$= -5(-5)^2 + 127$$
$$= 2$$

The firework was 2 m above the ground when it was fired at 0 s.

Chapter 4 Section 4 **Question 15** **Page 187**

a)



b) One point on the graph is (25, 0) representing an endpoint of the parabolic mirror. Substitute h = 0 and k = –0.24. Then, substitute x = 25 and y = 0 and solve for a.

$$
\begin{aligned}
y &= a(x-h)^2 + k \\
y &= a(x-0)^2 + (-0.24) \\
0 &= a(25)^2 - 0.24 \\
0.24 &= 625a \\
0.00038 &= a
\end{aligned}
$$

An equation that represents the cross section of the mirror is $y = 0.00038x^2 - 0.24$. It is valid for $-25 \le x \le 25$.

c) Answers will vary. For example:

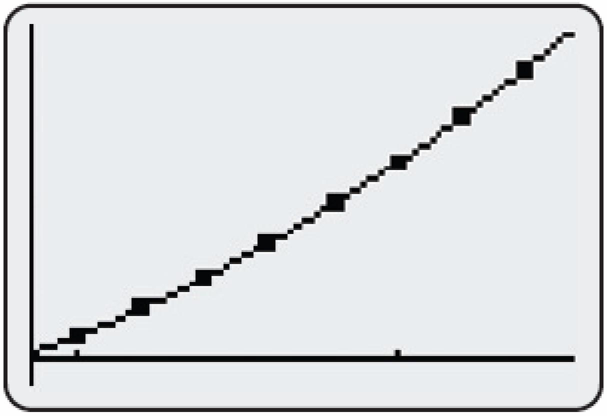
Place the vertex at the origin. Then, one point on the graph is (25, 0.24) representing an endpoint of the parabolic mirror. Substitute h = 0 and k = 0. Then, substitute x = 25 and y = 0.24 and solve for a.

$$
\begin{aligned}
y &= a(x-h)^2 + k \\
y &= a(x-0)^2 + 0 \\
0.24 &= a(25)^2 \\
0.24 &= 625a \\
0.00038 &= a
\end{aligned}
$$

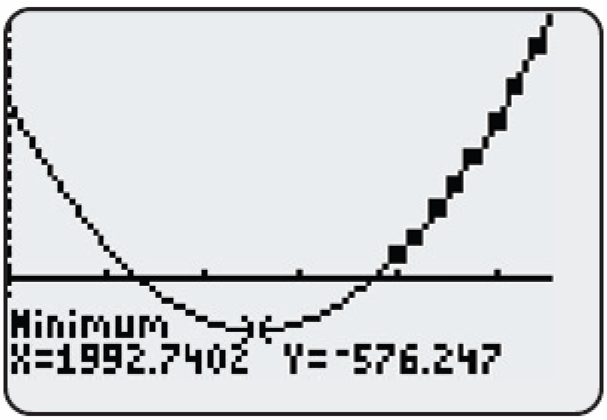
An equation that represents the cross section of the mirror is $y = 0.00038x^2$. It is valid for $-25 \le x \le 25$.

Chapter 4 Section 4 Question 16 Page 187

a)



b)



c) The curve of best fit is shown in part b). Substitute h = 1992.7 and k = –576.2. Then, substitute x = 2004 and y = 1292 and solve for a.

$$y = a(x-h)^2 + k$$

$$y = a(x-1992.7)^2 + (-576.2)$$

$$1292 = a(2004-1992.7)^2 - 576.2$$

$$1868.2 = 127.69a$$

$$14.6 \approx a$$

An equation for the curve of best fit is $y = 14.6(x-1992.7)^2 - 576.2$.

d) The equation is valid for $x \geq 2000$ and $y \geq 200$.

Chapter 4 Section 4 Question 17 Page 188

Left parabola: The vertex is (–100, 0) and one endpoint is (–50, 30).
Substitute h = –100 and k = 0. Then, substitute x = –50 and y = 30 and solve for a.

$$y = a(x-h)^2 + k$$
$$y = a(x-(-100))^2 + 0$$
$$30 = a(-50+100)^2$$
$$30 = 2500a$$
$$0.012 = a$$

An equation for the left parabola is $y = 0.012(x+100)^2$. This equation is valid for $-100 \le x \le -50$.

Middle parabola: The vertex is (0, 0) and one endpoint is (–50, 30).
Substitute h = 0 and k = 0. Then, substitute x = –50 and y = 30 and solve for a.

$$y = a(x-h)^2 + k$$
$$y = a(x-0)^2 + 0$$
$$30 = a(-50)^2$$
$$30 = 2500a$$
$$0.012 = a$$

An equation for the middle parabola is $y = 0.012x^2$. This equation is valid for $-50 \le x \le 50$.

Right parabola: The vertex is (100, 0) and one endpoint is (50, 30).
Substitute h = 100 and k = 0. Then, substitute x = 50 and y = 30 and solve for a.

$$y = a(x-h)^2 + k$$
$$y = a(x-100)^2 + 0$$
$$30 = a(50-100)^2$$
$$30 = 2500a$$
$$0.012 = a$$

An equation for the right parabola is $y = 0.012(x-100)^2$. This equation is valid for $50 \le x \le 100$.

Chapter 4 Section 4 Question 18 Page 188

Solutions for Achievement Checks are shown in the Teacher’s Resource.

Chapter 4 Section 4 Question 19 Page 188

a) The transformed equation is $y = -2(x-4)^2 + 1$.

b) The transformed equation is $y = 2x^2 - 1$.

c) The transformed equation is $y = -2(x-4)^2 + 4$.

d) The transformed equation is $y = 2(x+4)^2 - 1$.

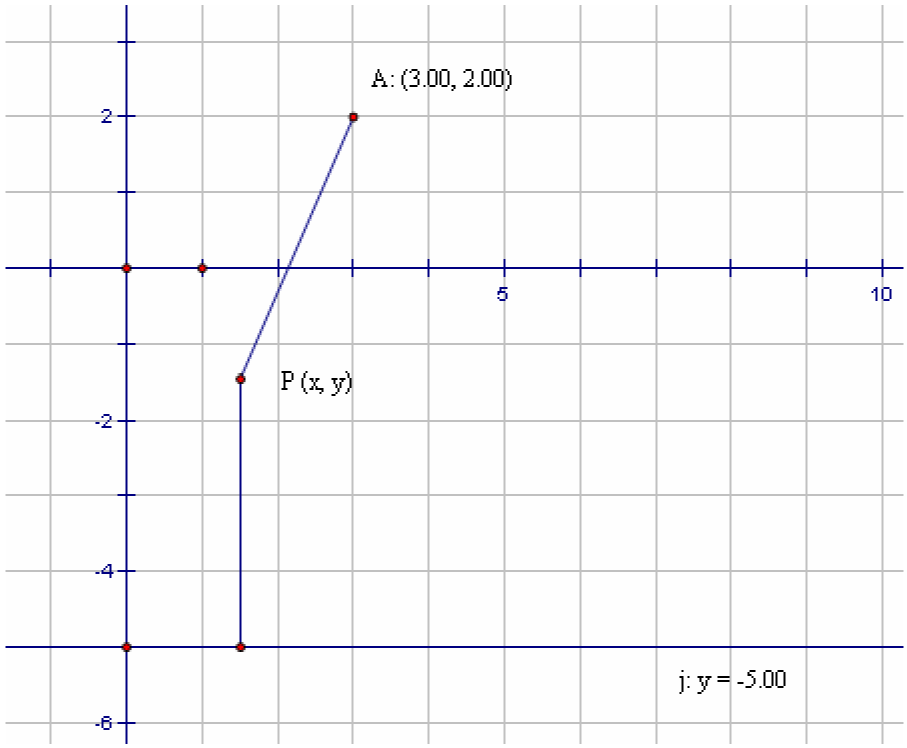
Chapter 4 Section 4 Question 20 Page 188

a) radius 5, centred at (0, 3): $x^2 + (y-3)^2 = 25$
radius 7, centred at (6, 1): $(x-6)^2 + (y-1)^2 = 49$
radius 8, centred at (–3, 5): $(x+3)^2 + (y-5)^2 = 64$
radius *r*, centred at (*h*, *k*): $(x-h)^2 + (y-k)^2 = r^2$

b) Answers will vary. For example:

A circle with equation $(x-h)^2 + (y-k)^2 = r^2$ has centre (*h*, *k*). A parabola with equation $y = (x-h)^2 + k$ has vertex (*h*, *k*).

Chapter 4 Section 4 Question 21 Page 188



Let P(x, y) be any point that is equidistant from the point A(3, 2) and the line $y = -5$.

Equate the distances, and solve for y.

$$\begin{aligned}\sqrt{(x-3)^2+(y-2)^2} &= \sqrt{(x-x)^2+(y-(-5))^2}\\ (x-3)^2+(y-2)^2 &= (y+5)^2\\ x^2-6x+9+y^2-4y+4 &= y^2+10y+25\\ -14y &= -x^2+6x+12\\ y &= \frac{1}{14}x^2-\frac{3}{7}x-\frac{6}{7}\end{aligned}$$

The equation for the required locus is $y = \frac{1}{14}x^2 - \frac{3}{7}x - \frac{6}{7}$.

Chapter 4 Section 4 **Question 22** **Page 188**

$$a + b = 21$$

$$\frac{1}{a} + \frac{1}{b} = \frac{7}{18}$$

$$\frac{a+b}{ab} = \frac{7}{18}$$

$$\frac{21}{ab} = \frac{7}{18}$$

$$ab = \frac{21 \times 18}{7}$$

$$ab = 54$$

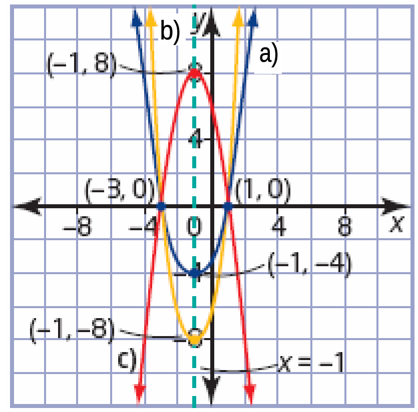
Answer **C**

Chapter 4 Section 5 Quadratic Relations of the Form $y = a(x - r)(x - s)$

Chapter 4 Section 5 Question 1 Page 192

The three graphs all have the same *x*-intercepts and axis of symmetry.

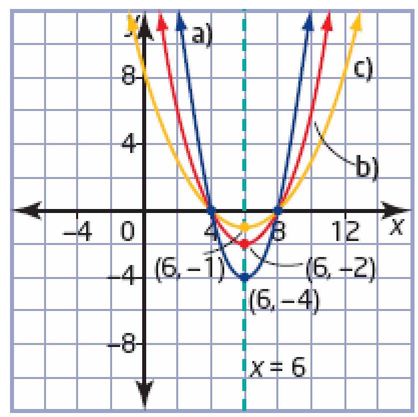
The graphs differ in the vertical stretch of the parabola, and the direction of opening.



Chapter 4 Section 5 Question 2 Page 192

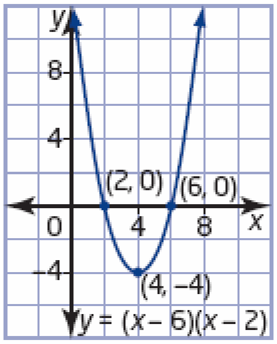
The three graphs all have the same *x*-intercepts, axis of symmetry, and direction of opening.

The graphs differ in the vertical stretch of the parabola.

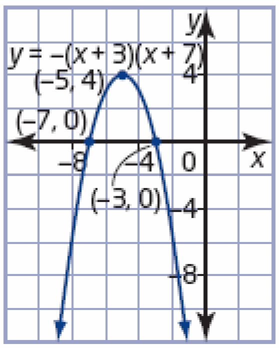


Chapter 4 Section 5 **Question 3 Page 192**

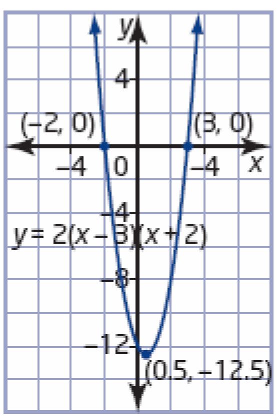
a)



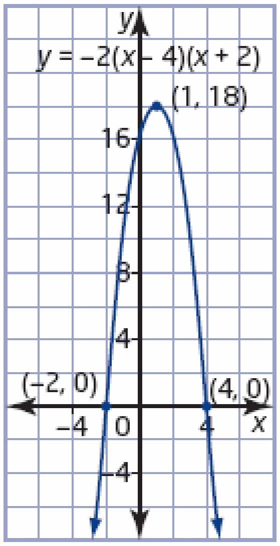
b)



c)

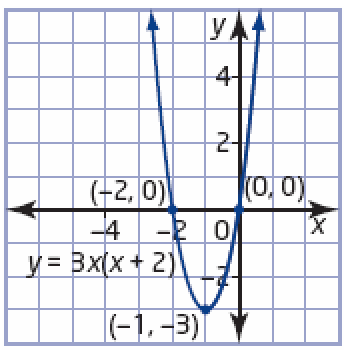


d)

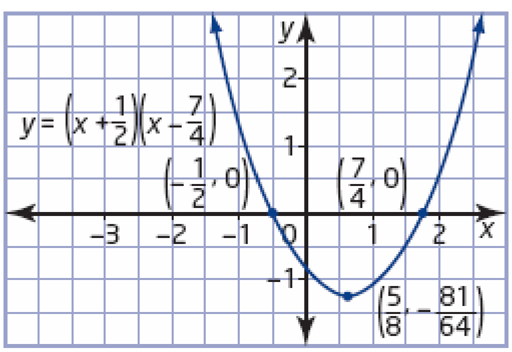


Chapter 4 Section 5 **Question 4** **Page 192**

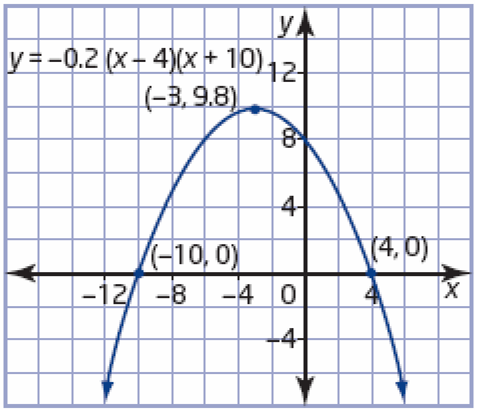
a)



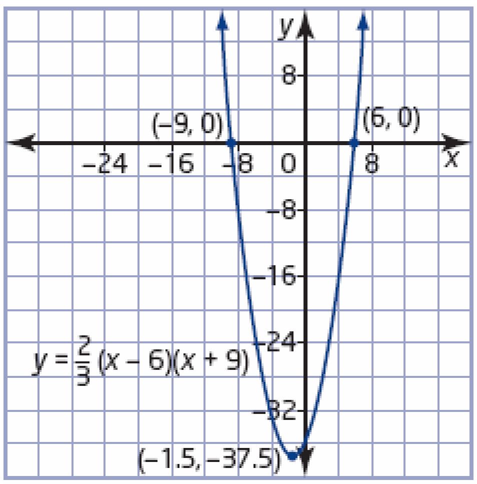
b)



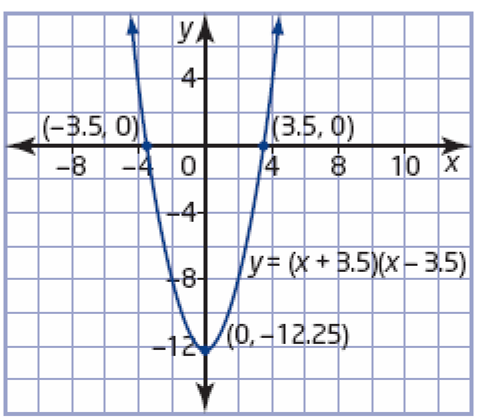
c)


y = –0.2 (x – 4)(x + 10)
(–3, 9.8)
(–10, 0)
(4, 0)
12
8
4
–4
–12
–8
–4
0
4
x
y

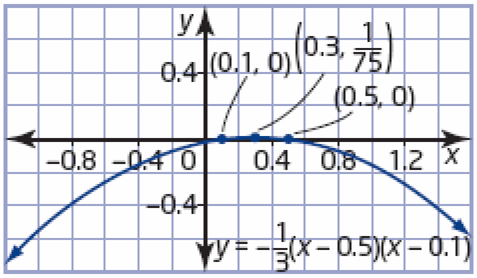
d)


(–9, 0)
(6, 0)
y = 2/3 (x – 6)(x + 9)
(–1.5, –37.5)
8
–8
–16
–24
–32
–24
–16
–8
0
8
x
y

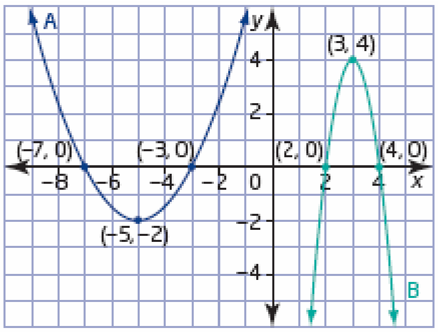
e)


(–3.5, 0)
(3.5, 0)
y = (x + 3.5)(x – 3.5)
(0, –12.25)
4
–4
–8
–12
–8
–4
0
4
8
10
x
y

f)



Chapter 4 Section 5 Question 5 Page 192



The x-intercepts for the left parabola are –7 and –3. So, the equation has the form $y = a(x + 7)(x + 3)$. Substitute the coordinates of the vertex and solve for a.

$$\begin{aligned}-2 &= a(-5+7)(-5+3)\\ -2 &= a(2)(-2)\\ -2 &= -4a\\ 0.5 &= a\end{aligned}$$

An equation for the parabola is $y = 0.5(x+7)(x+3)$.

The x-intercepts for the right parabola are 2 and 4. So, the equation has the form $y = a(x - 2)(x - 4)$. Substitute the coordinates of the vertex and solve for a.

$$\begin{aligned}y &= a(x-r)(x-s)\\ &= a(x-2)(x-4)\\ 4 &= a(3-2)(3-4)\\ 4 &= a(1)(-1)\\ 4 &= -a\\ -4 &= a\end{aligned}$$

An equation for the parabola is $y = -4(x-2)(x-4)$.

Chapter 4 Section 5 Question 6 Page 192

a) The vertex is (5, 0).

b) The parabola has one *x*-intercept.

c) $y = (x - 5)(x - 5)$

Chapter 4 Section 5 Question 7 Page 192

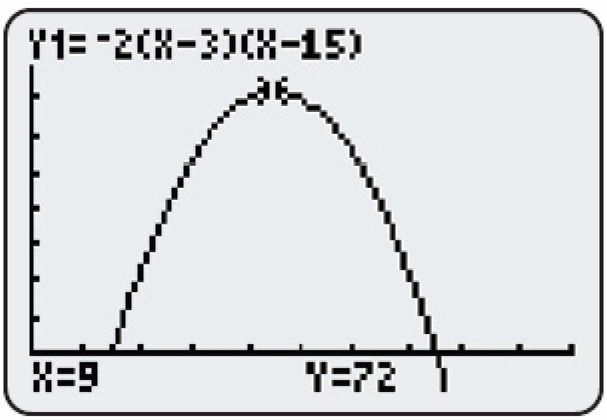
a) $y = (x + 2)^2$

The parabola has an *x*-intercept at –2.

b) The vertex is at the *x*-intercept. The coordinates are (–2, 0).

Chapter 4 Section 5 Question 8 Page 192

a)



b) $h = -2(d - 3)(d - 15)$

$h = 0$ when $d = 0$ or $d = 15$. The first value is the launch point. The second is the landing point. The rocket lands 15 m from the safety wall.

c) The vertex is halfway between the launch and landing. This value is $\frac{3+15}{2}$, or 9 m from the wall.

$$\begin{aligned} h &= -2(d-3)(d-15) \\ &= -2(9-3)(9-15) \\ &= -2(6)(-6) \\ &= 72 \end{aligned}$$

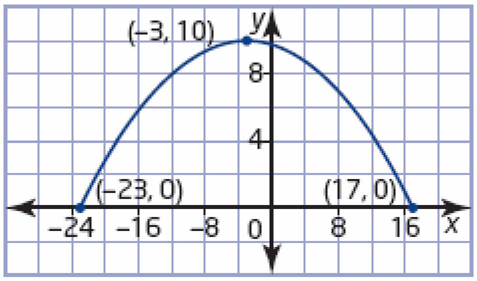
The rocket reached its maximum height of 72 m at a horizontal distance of 9 m from the safety wall.

Chapter 4 Section 5 Question 9 Page 193

When the *x*-intercepts are opposite values, the vertex has an *x*-coordinate of 0.

Chapter 4 Section 5 Question 10 Page 193

a)



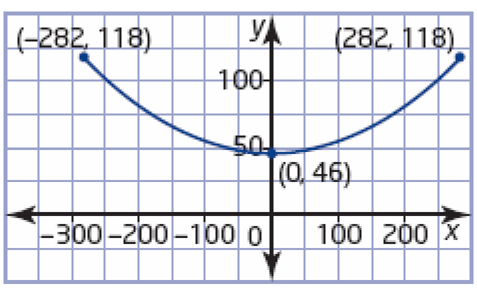
b) The *x*-intercepts are –23 and 17. So, the equation has the form $y = a(x + 23)(x - 17)$. Substitute the coordinates of the vertex and solve for *a*.

$$10 = a(-3 + 23)(-3 - 17)$$
$$10 = a(20)(-20)$$
$$10 = -400a$$
$$-\frac{1}{40} = a$$

An equation for the path of the soccer ball is $y = -\frac{1}{40}(x+23)(x-17)$.

Chapter 4 Section 5 Question 11 Page 193

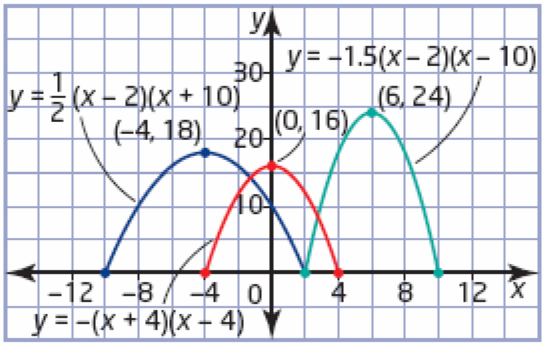
a)



b) It is not possible to write an equation in the form $y = a(x-r)(x-s)$. The graph has no *x*-intercepts.

Chapter 4 Section 5 Question 12 Page 193

a)



b) The base of the museum runs from $x = -10$ to $x = 10$, for a width of 20 m.

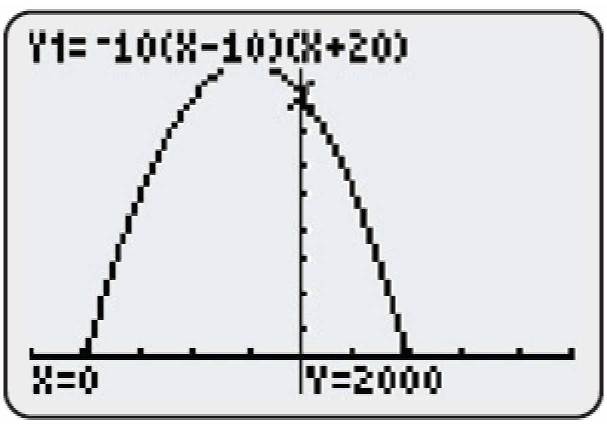
c) The blue arch is 18 m tall. The red arch is 16 m tall. The green arch is 24 m tall.

Chapter 4 Section 5 Question 13 Page 193

a) $R = (100 - 10x)(20 + x)$

$= -10(x - 10)(x + 20)$

b)



c) The R-intercept represents the revenue with the current ticket price of \$20. The x-intercepts represent the number of price increases or decreases that would result in revenue of \$0.

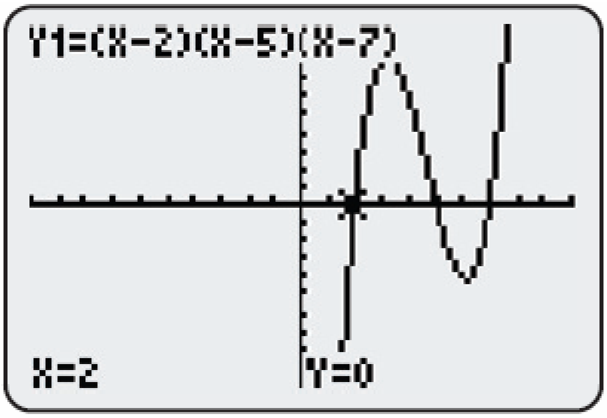
d) A negative value of x represents a decrease in ticket price.

e) The vertex occurs at an x-coordinate of $\frac{-20+10}{2}$, or –5.

The ticket price for maximum revenue is 20 – 5, or \$15.

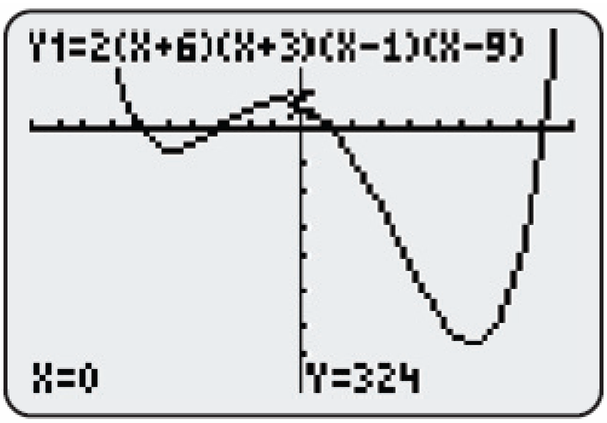
Chapter 4 Section 5 Question 14 Page 193

a)



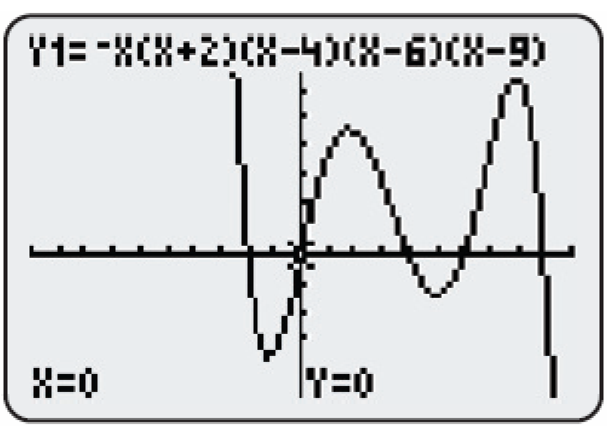
The equation of the relation has three factors. The graph of the relation crosses the *x*-axis at three points. The three points are the *x*-intercepts of the relation.

b)



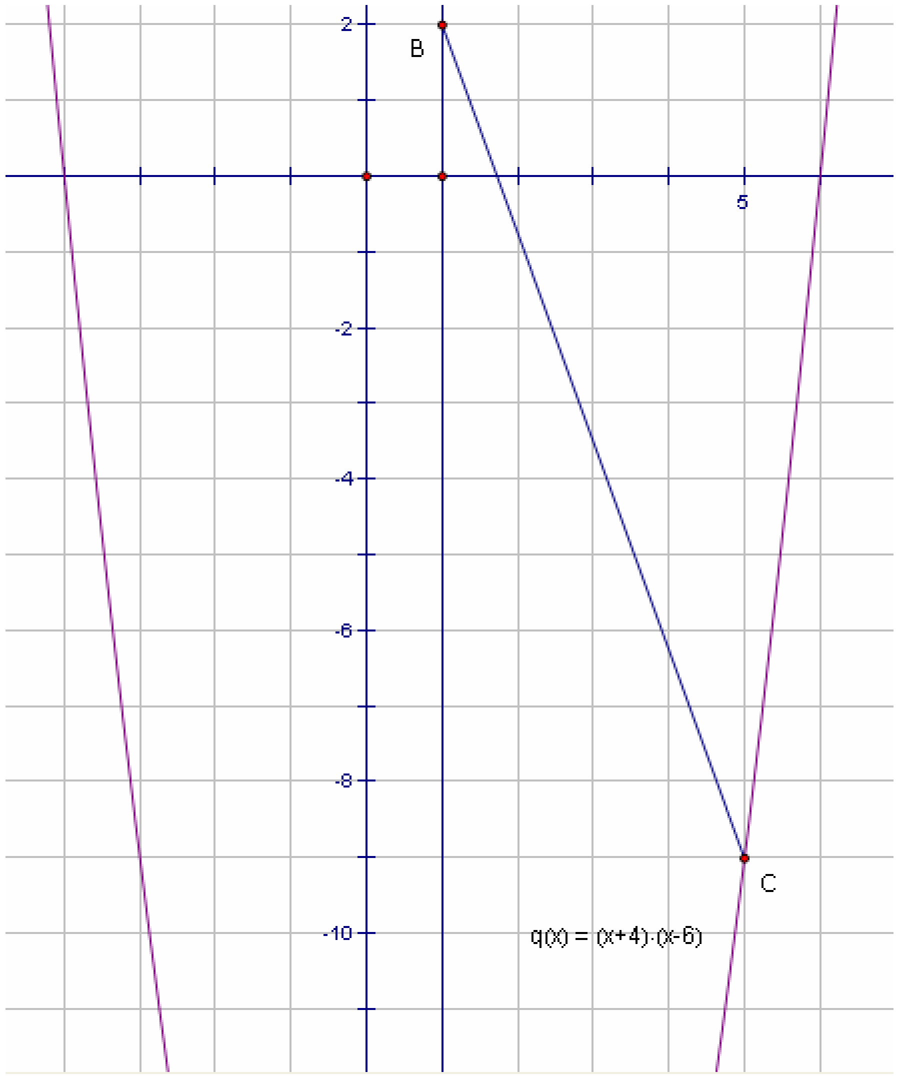
The equation of the relation has four factors. The graph of the relation crosses the *x*-axis at four points. The four points are the *x*-intercepts of the relation.

c)



The equation of the relation has five factors. The graph of the relation crosses the *x*-axis at five points. The five points are the *x*-intercepts of the relation.

Chapter 4 Section 5 **Question 15** **Page 193**



The *x*-intercepts are –4 and 6. The *x*-coordinate of the vertex is $\frac{-4+6}{2}$, or 1.

The axis of symmetry for the parabola is $x = 1$. Then, the coordinates of point B are (1, 2).

Find the coordinates of point C.

$$\begin{aligned} y &= (x+4)(x-6) \\ &= (5+4)(5-6) \\ &= -9 \end{aligned}$$

The coordinates of point C are (5, –9).

$$
\begin{aligned}
\text{BC} &= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \\
&= \sqrt{\left(5 - 1\right)^2 + \left(-9 - 2\right)^2} \\
&= \sqrt{4^2 + \left(-11\right)^2} \\
&= \sqrt{137}
\end{aligned}
$$

The distance from B to C is $\sqrt{137}$ units.

Chapter 4 Section 6 — Negative and Zero Exponents

Chapter 4 Section 6 — Question 1 Page 199

a) $3^{-2}=\frac{1}{3^2}$

b) $5^{-1}=\frac{1}{5^1}$

c) $10^{-4}=\frac{1}{10^4}$

d) $7^{-3}=\frac{1}{7^3}$

e) $(-2)^{-4}=\frac{1}{(-2)^4}$

f) $(-7)^{-1}=\frac{1}{(-7)^1}$

Chapter 4 Section 6 — Question 2 Page 199

a) $6^{-2}=\frac{1}{36}$

b) $9^0=1$

c) $7^{-1}=\frac{1}{7}$

d)
$$\begin{aligned}10^{-3}&=\frac{1}{10^3}\\&=\frac{1}{1000}\end{aligned}$$

e)
$$\begin{aligned}(-9)^{-1}&=\frac{1}{(-9)^1}\\&=-\frac{1}{9}\end{aligned}$$

f)
$$\begin{aligned}(-12)^{-2}&=\frac{1}{(-12)^2}\\&=\frac{1}{144}\end{aligned}$$

g) $(-3)^0=1$

h) $-89^0=-1$

Chapter 4 Section 6 — Question 3 Page 199

a)
$$\begin{aligned}\left(\frac{1}{3}\right)^{-2}&=\frac{1}{\left(\frac{1}{3}\right)^2}\\&=\frac{1}{\frac{1}{9}}\\&=1\times\frac{9}{1}\\&=9\end{aligned}$$

b) 0^{-5} is undefined.

c)
$$\begin{aligned}\left(-\frac{1}{4}\right)^{-1}&=\frac{1}{\left(-\frac{1}{4}\right)^1}\\&=\frac{1}{-\frac{1}{4}}\\&=1\times\left(-\frac{4}{1}\right)\\&=-4\end{aligned}$$

d)
$$\begin{aligned}\left(\frac{5}{6}\right)^{-2}&=\frac{1}{\left(\frac{5}{6}\right)^2}\\&=\frac{1}{\frac{25}{36}}\\&=\frac{36}{25}\end{aligned}$$

e)
$$\begin{aligned}\left(-\frac{3}{8}\right)^{-4}&=\frac{1}{\left(-\frac{3}{8}\right)^4}\\&=\frac{1}{\frac{81}{4096}}\\&=\frac{4096}{81}\end{aligned}$$

f)
$$\begin{aligned}\left(\frac{9}{4}\right)^{-3}&=\frac{1}{\left(\frac{9}{4}\right)^3}\\&=\frac{1}{\frac{729}{64}}\\&=\frac{64}{729}\end{aligned}$$

Chapter 4 Section 6 Question 4 Page 199

a) $6^0 + 6^{-2} = 1 + \frac{1}{36}$
$= 1\frac{1}{36}$

b) $8 - 8^{-1} = 8 - \frac{1}{8}$
$= 7\frac{7}{8}$

c) $(4+3)^0 = 1$

d) $4^0 + 3^0 = 1 + 1$
$= 2$

Chapter 4 Section 6 Question 5 Page 199

a) 52 h is 4 half-lives. The fraction that remains is $\left(\frac{1}{2}\right)^4$, or $\frac{1}{16}$.

b) 78 h is 6 half-lives. The fraction that remains is $\left(\frac{1}{2}\right)^6$, or $\frac{1}{64}$.

c) $\left(\frac{1}{2}\right)^4 = 2^{-4}$ $\left(\frac{1}{2}\right)^6 = 2^{-6}$

Chapter 4 Section 6 Question 6 Page 199

a) 9 billion years is 2 half-lives. The mass that remains is $0.5 \times \left(2^{-1}\right)^2$, or 0.125 kg.

b) 22.5 billion years is 5 half-lives. The mass that remains is $0.5 \times \left(2^{-1}\right)^5$, or 0.015 625 kg.

Chapter 4 Section 6 Question 7 Page 199

a) $\frac{1}{16} = 2^{-4}$

b) The remaining mass of radium-226 after 6400 years is $8 \times \frac{1}{16}$, or 0.5 mg.

Chapter 4 Section 6 Question 8 Page 199

a) $3^{-3} = \frac{1}{3^3}$

$= \frac{1}{27}$

$x = 3$

b) $\left(\frac{5}{4}\right)^{-1} = \frac{4}{5}$

$x = \frac{5}{4}$

c) $2^{-2} = \frac{1}{2^2}$

$= \frac{1}{4}$

$x = –2$

d) $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3}$

$= \frac{125}{8}$

$x = –3$

Chapter 4 Section 6 Question 9 Page 199

Answers will vary.

Chapter 4 Section 6 Question 10 Page 199

Answers will vary.

Chapter 4 Section 6 Question 11 Page 200

a)

$$N = 1000 \times 2^t \quad N = 1000 \times 2^t \quad N = 1000 \times 2^t \quad N = 1000 \times 2^t$$
$$= 1000 \times 2^2 \quad = 1000 \times 2^3 \quad = 1000 \times 2^4 \quad = 1000 \times 2^5$$
$$= 4000 \quad = 8000 \quad = 16\,000 \quad = 32\,000$$

The number of bees after 2, 3, 4, and 5 months are, respectively, 4000, 8000, 16 000, and 32 000.

b) $t = 0$ represents June 1.

c) $t = –1$ represents one month before June 1, or May 1.

d)

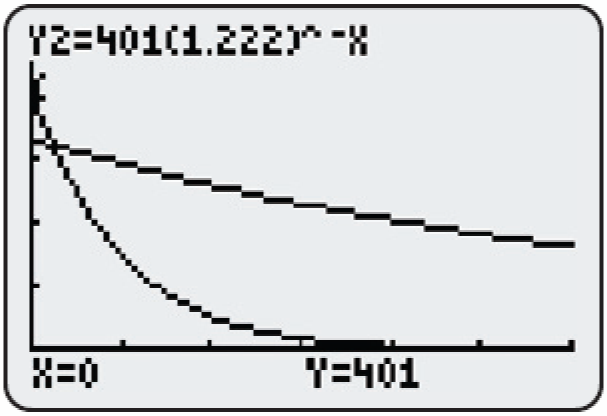
$$N = 1000 \times 2^t$$
$$125 = 1000 \times 2^t$$
$$\frac{125}{1000} = 2^t$$
$$\frac{1}{8} = 2^t$$
$$2^{-3} = 2^t$$

There were 125 bees 3 months before June 1, or March 1.

Chapter 4 Section 6 Question 12 Page 200

a) A negative exponent is used because the intensity of the light energy is decreasing.

b)



c) The intensity decreases more quickly in Lake Erie. The base is greater.

Chapter 4 Section 6 Question 13 Page 200

Solutions for Achievement Checks are shown in the Teacher's Resource.

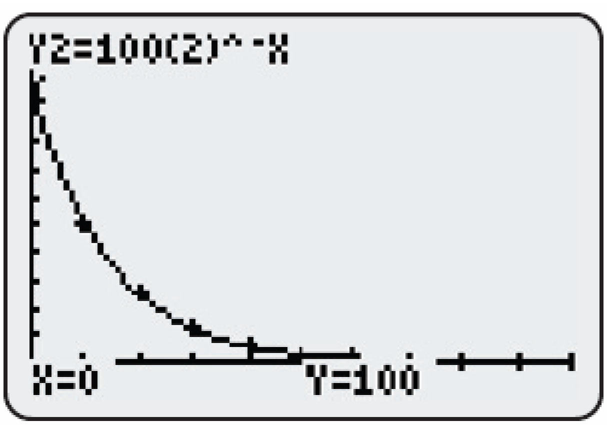
Chapter 4 Section 6 Question 14 Page 200

a) After 10 minutes, Chris has walked 50 + 25 + 12.5 + 6.25 + 3.125 + 1.5625 + 0.781 25 + 0.390 625 + 0.195 3125 + 0.097 656 25 = 99.902 343 75 m.

b) Chris will never get to the end of the track. He will always be walking a distance of half the previous distance. Looking at the graph, the curve will never reach zero.

Time (min)	Distance Remaining (m)
1	50
2	25
3	12.5
4	6.25
5	3.125
6	1.562 5
7	0.781 25
8	0.390 625
9	0.195 312 5
10	0.097 656 25

c) The distance remaining after *t* minutes can be modelled using the equation $d = 100(2)^{-t}$.



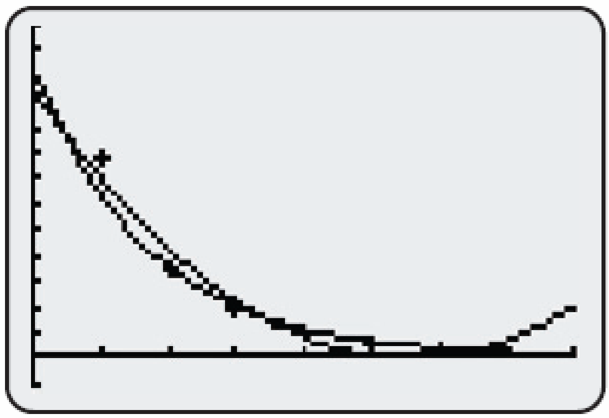
Chapter 4 Section 6 **Question 15** **Page 201**

a) The equation is $m = 500(0.9)^t$.

b) Use a calculator to try various values of *t*. 1% of the original mass is 5 g. Less than 1% of the original mass will remain after 44 h.

Chapter 4 Section 6 **Question 16** **Page 201**

a)–c)

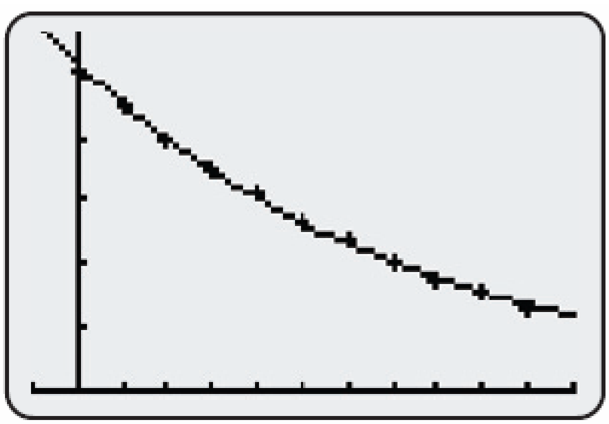


Weekend in 2006	Ticket Sales ($millions)
May 19–May 21	77.1
May 26–May 28	34.0
June 2–June 4	18.6
June 9–June 11	10.4
June 16–June 18	5.3
June 23–June 25	4.1
June 30–July 2	2.3

d) The exponential model fits the data better.

Chapter 4 Section 6 **Question 17** **Page 201**

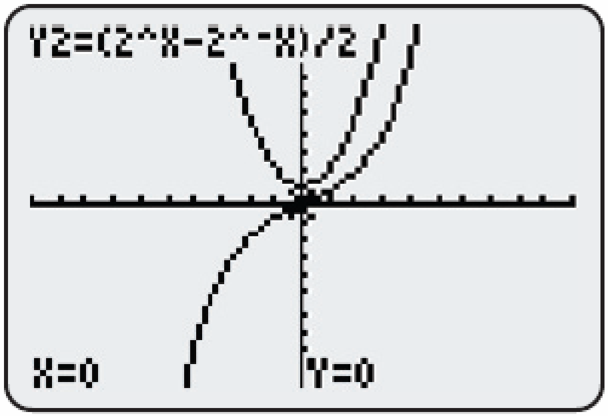
a), b)



Altitude (km)	Pressure (millibars)
0	1013.3
1	898.8
2	795.0
3	701.3
4	616.5
5	540.5
6	472.2
7	411.1
8	356.5
9	307.9
10	265.0

An exponential model is better than a quadratic model because the atmospheric pressure never reaches 0 millibars.

Chapter 4 Section 6 Question 18 Page 201



The graph of $y = x^2 + 1$ is a parabola with vertex (0, 1), opening upward. When $x < 0$, the graph of $y = \frac{2^x - 2^{-x}}{2}$ looks like a parabola that opens downward and is wider than the parabola $y = x^2 + 1$. When $x > 0$, the graph of $y = \frac{2^x - 2^{-x}}{2}$ looks like a parabola that opens upward and is wider than the parabola $y = x^2 + 1$. The graph of $y = \frac{2^x - 2^{-x}}{2}$ crosses the *y*-axis at the origin and changes direction at this point.

Chapter 4 Section 6 Question 19 Page 201

a)

$$\begin{aligned} 3^x &= \frac{1}{81} \\ &= \frac{1}{3^4} \\ &= 3^{-4} \end{aligned}$$

$x = -4$

b) $$\begin{aligned} 4\left(2^{3x}\right) &= \frac{1}{16} \\ 2^2\left(2^{3x}\right) &= 2^{-4} \\ 2^{2+3x} &= 2^{-4} \end{aligned}$$

Set the expressions for the exponents equal and solve for *x*.

$$\begin{aligned} 2 + 3x &= -4 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

Chapter 4 Section 6 Question 20 Page 201

a) The greatest possible value is 13.

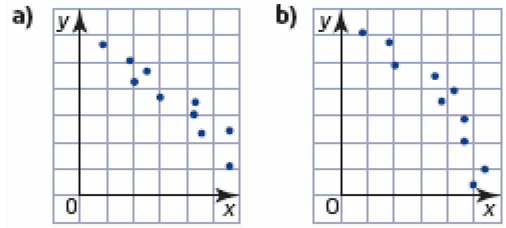
$$
\begin{aligned}
a^b + c^d + e^f + g &= (-3)^2 + (-1)^{-2} + (3)^1 + 0 \\
&= 13
\end{aligned}
$$

b) The least possible value is –30.

$$
\begin{aligned}
a^b + c^d + e^f + g &= (-3)^3 + (-2)^1 + (0)^2 + (-1) \\
&= -30
\end{aligned}
$$

Chapter 4 Review

Chapter 4 Review **Question 1** **Page 202**

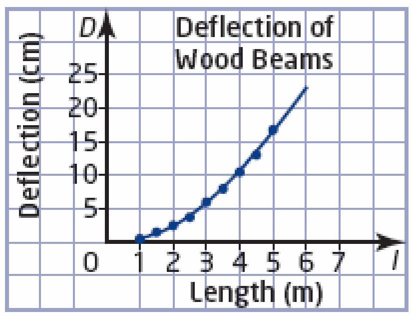


The scatter plot in b) could be modelled using a curve. The points lie on a curve.

Chapter 4 Review **Question 2** **Page 202**

a)

Length (m)	Deflection (cm)
1.0	0.33
1.5	1.48
2.0	2.51
2.5	4.22
3.0	6.11
3.5	8.17
4.0	10.52
4.5	12.98
5.0	16.72



b) The points lie along a curve. The relation between the variables is non-linear.

c) The deflection of a 6.0-m-long beam is about 23.5 cm.

Chapter 4 Review **Question 3** **Page 202**

a)

x	*y*	First Differences	Second Differences
1	11		
2	18	7	
3	27	9	2
4	38	11	2
5	51	13	2

The second differences are constant. The relation is quadratic.

b)

x	*y*	First Differences	Second Differences
–2	–10		
–1	–2	8	
0	0	2	–6
1	2	2	0
2	10	8	6

Neither the first differences nor the second difference are constant. The relation is neither linear nor quadratic.

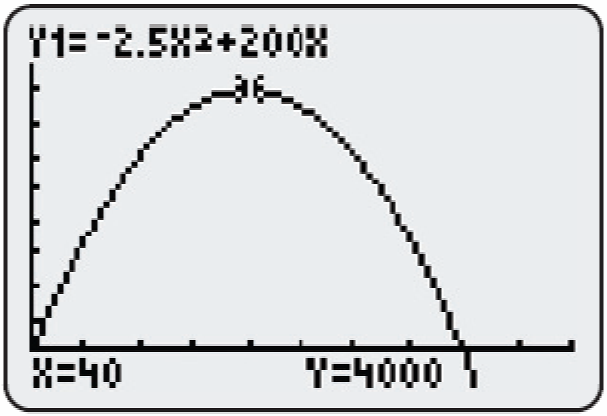
c)

x	*y*	First Differences
–2	–-9	
–1	–6	3
0	–3	3
1	0	3
2	3	3

The first differences are constant. The relation is linear.

Chapter 4 Review **Question 4 Page 202**

a)



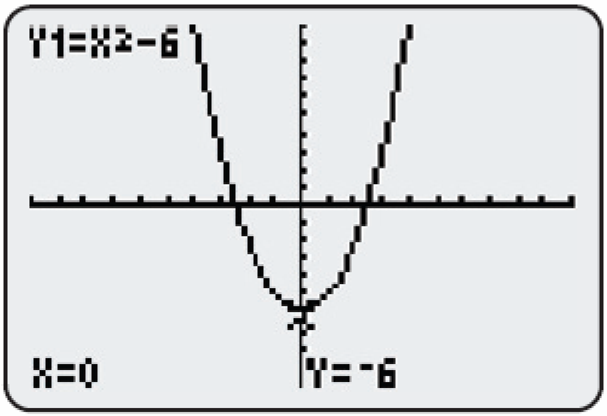
b) Read the value from your graph or table of values or use the TRACE and ZOOM features on the graphing calculator. The curve crosses the *x*-axis at 0 and 80.

It takes 80 min to fly from Toronto to Montréal.

c) Read the value from your graph or table of values or use the TRACE and ZOOM features on the graphing calculator. The aircraft reaches its maximum height of 4000 m halfway through the flight, at 40 min.

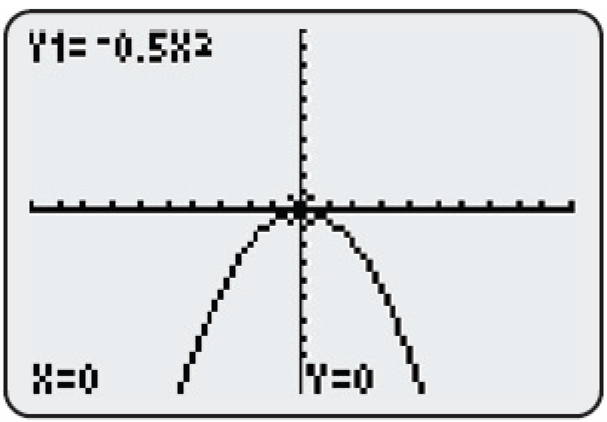
Chapter 4 Review **Question 5** **Page 202**

a)



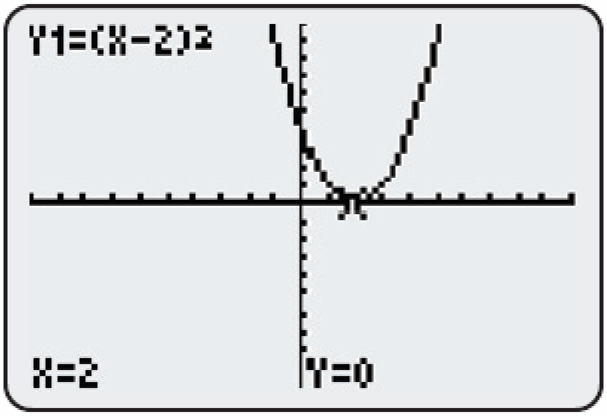
The graph of $y = x^2 - 6$ is the graph of $y = x^2$ translated 6 units downward.

b)



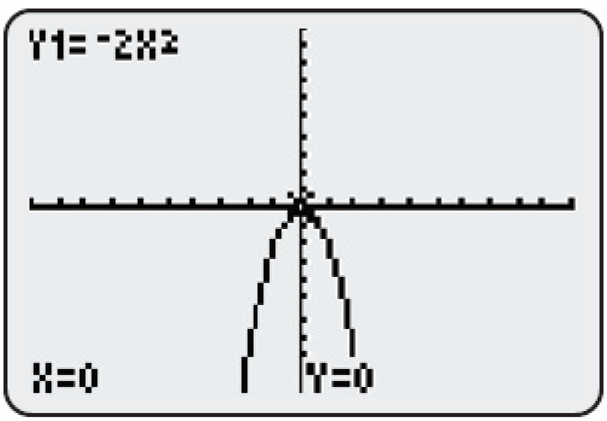
The graph of $y = -0.5x^2$ is the graph of $y = x^2$ reflected in the *x*-axis and compressed vertically by a factor of 0.5.

c)



The graph of $y=(x-2)^2$ is the graph of $y=x^2$ translated 2 units to the right.

d)



The graph of $y=-2x^2$ is the graph of $y=x^2$ reflected in the *x*-axis and stretched vertically by a factor of 2.

a)

Property	$y = (x - 1)^2 - 4$
Vertex	(1, −4)
Axis of symmetry	$x = 1$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -4$

b)

Property	$y = 2(x + 3)^2 + 1$
Vertex	(−3, 1)
Axis of symmetry	$x = -3$
Stretch or compression factor relative to $y = x^2$	2
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 1$

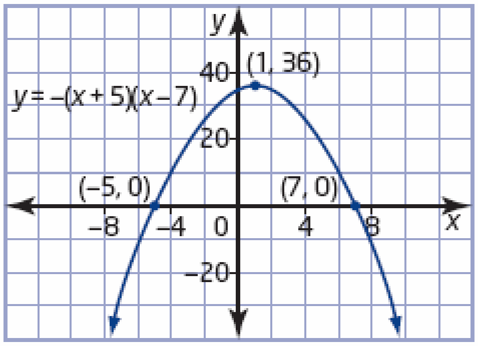
c)

Property	$y = \frac{1}{4}(x - 5)^2 + 1$
Vertex	(5, 1)
Axis of symmetry	$x = 5$
Stretch or compression factor relative to $y = x^2$	$\frac{1}{4}$
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 1$

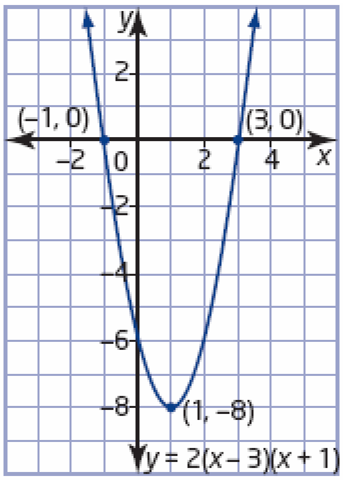
d)

Property	$y = -(x + 2)^2 + 6$
Vertex	(−2, 6)
Axis of symmetry	$x = -2$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq 6$

a)

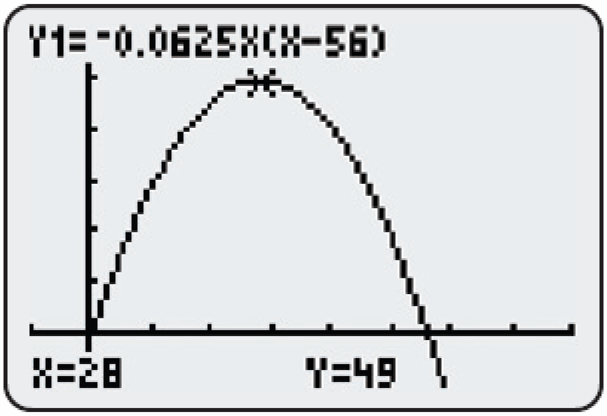


b)



Chapter 4 Review **Question 8** **Page 203**

a)



b) $h = -0.0625d(d-56)$

h = 0 at d = 0 and d = 56. The ball lands at a horizontal distance of 56 m.

c) The football reaches its maximum height halfway between the kick and the landing at $\frac{0+56}{2}$, or 28 m.

$$\begin{aligned} h &= -0.0625d(d-56) \\ &= -0.0625(28)(28-56) \\ &= 49 \end{aligned}$$

The football reaches a maximum height of 49 m.

Chapter 4 Review **Question 9 Page 203**

a) $7^{-2} = \frac{1}{7^2}$
$= \frac{1}{49}$

b) $13^0 = 1$

c) $10^{-5} = \frac{1}{10^5}$
$= \frac{1}{100\,000}$

d) $(-34)^0 = 1$

e) $(-6)^{-1} = \frac{1}{(-6)^1}$
$= -\frac{1}{6}$

f) $(-7)^{-2} = \frac{1}{(-7)^2}$
$= \frac{1}{49}$

g) $6^0 = 1$

h) $\left(-\frac{2}{5}\right)^{-3} = \frac{1}{\left(-\frac{2}{5}\right)^3}$
$= \frac{1}{-\frac{8}{125}}$
$= -\frac{125}{8}$

Chapter 4 Review **Question 10 Page 203**

a) $\left(\frac{1}{2}\right)^6 = \frac{1}{64}$

$\frac{1}{64}$ of the winnings remain after 6 months.

b) $\left(\frac{1}{2}\right)^{12} = \frac{1}{4096}$

$\frac{1}{4096}$ of the winnings remain after 12 months.

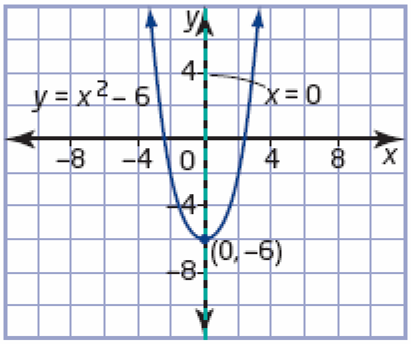
c) $\frac{1}{64} = 2^{-6}$ $\frac{1}{4096} = 2^{-12}$

d) The amount remaining at the end of the year is $\frac{1\,000\,000}{4096}$, or about \$244.14.

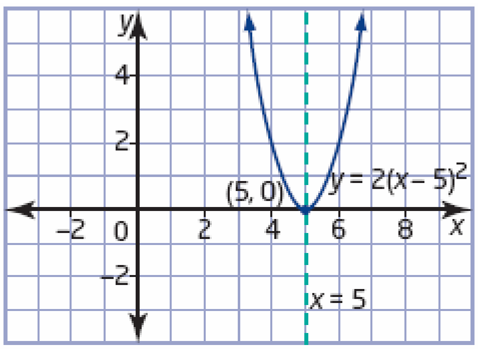
Chapter 4 Chapter Test

Chapter 4 Chapter Test Question 1 Page 204

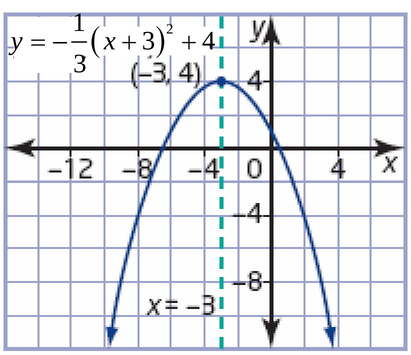
a)



b)

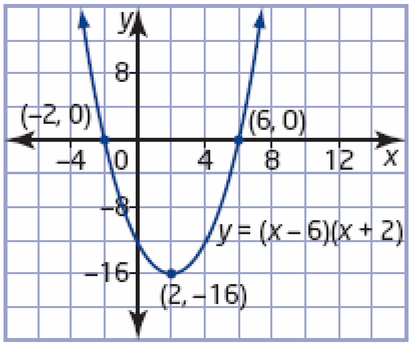


c)

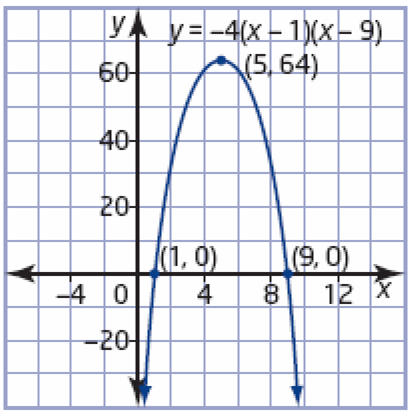


Chapter 4 Chapter Test Question 2 Page 204

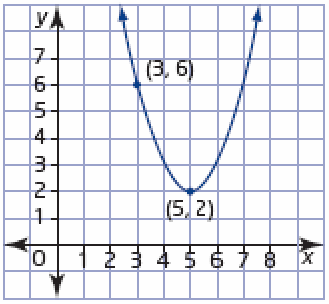
a)



b)



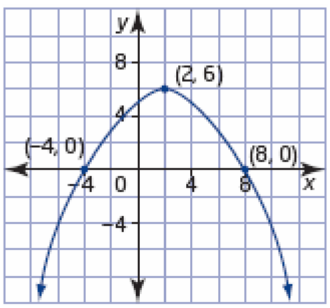
a)



The vertex is at (5, 2). There is no vertical stretch or compression.

An equation for the parabola is $y = (x - 5)^2 + 2$.

b)



The vertex is at (2, 6). There is a reflection in the *x*-axis, and a vertical compression. Substitute h = 2 and k = 6. Then, substitute x = 8 and y = 0 and solve for a.

$$y = a(x - h)^2 + k$$
$$y = a(x - 2)^2 + 6$$
$$0 = a(8 - 2)^2 + 6$$
$$-6 = 36a$$
$$-\frac{1}{6} = a$$

An equation for the parabola is $y = -\frac{1}{6}(x - 2)^2 + 6$.

Alternatively, an equation for the parabola in the form $y = a(x - r)(x - s)$ is

$y = -\frac{1}{6}(x + 4)(x - 8)$.

Chapter 4 Chapter Test Question 4 Page 204

a) $4^0 = 1$

b) $5^{-1} = \frac{1}{5^1}$
$= \frac{1}{5}$

c) $(-3)^{-3} = \frac{1}{(-3)^3}$
$= -\frac{1}{27}$

d) $\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2}$
$= \frac{1}{\frac{9}{16}}$
$= \frac{16}{9}$

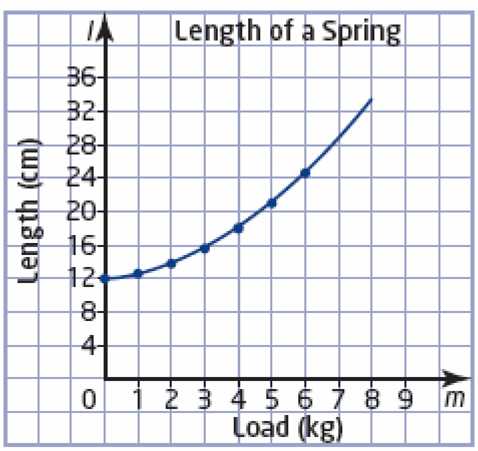
Chapter 4 Chapter Test Question 5 Page 204

a)

Load (kg)	Length (cm)	First Differences	Second Differences
0	12.0		
1	12.6	0.6	
2	13.8	1.2	0.6
3	15.6	1.8	0.6
4	18.0	2.4	0.6
5	21.0	3.0	0.6
6	24.6	3.6	0.6

The second differences are constant. The relation is quadratic.

b)



c) The length of the spring under a load of 8 kg is about 33.6 cm.

Chapter 4 Chapter Test Question 6 Page 204

a) $l = 0.011a^2 - 0.68a + 13.31$
$= 0.011(40)^2 - 0.68(40) + 13.31$
$= 3.71$

A 40-year-old tree will provide 371 board-feet of lumber when cut.

b) $l = 0.011a^2 - 0.68a + 13.31$
$= 0.011(75)^2 - 0.68(75) + 13.31$
$= 24.185$

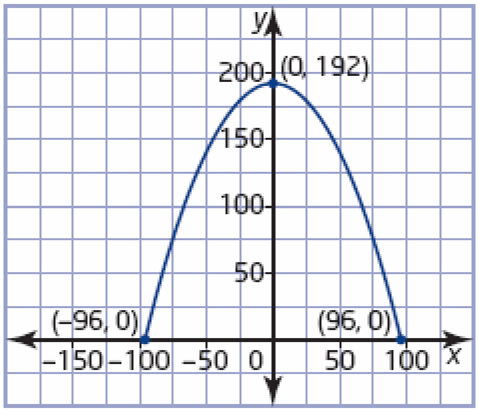
A 75-year-old tree will provide 2418.5 board-feet of lumber when cut.

c) Answers will vary. For example: The number of board feet increases in a quadratic manner because the cross-sectional area of the tree increases in a quadratic manner.

d) Answer will vary. For example: Forest rangers and lumber companies would be interested in knowing this information.

Chapter 4 Chapter Test Question 7 Page 205

a), **b)**



c) The vertex is at (0, 192). Substitute $h = 0$ and $k = 192$. Then, substitute $x = 96$ and $y = 0$ and solve for a.

$$y = a(x-h)^2 + k$$
$$y = a(x-0)^2 + 192$$
$$0 = a(96)^2 + 192$$
$$-192 = 9216a$$
$$-\frac{1}{48} = a$$

An equation that models the arch is $y = -\frac{1}{48}x^2 + 192$.

Chapter 4 Chapter Test Question 8 Page 205

Answers will vary. For example:

If a car uses tires with better grip, the minimum turn radius will decrease. The value of a will be less than 0.6.

Chapter 4 Chapter Test Question 9 Page 205

a) $h = \frac{3}{40}d^2$

$= \frac{3}{40}(25)^2$

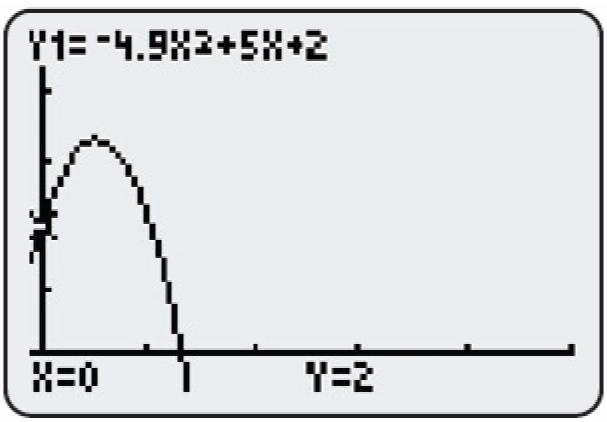
$= 46.875$

You need a height of 46.875 m above the surface of Earth to see 25 km.

b) If you were standing on a 20-m cliff, you would use the formula $h = \frac{3}{40}d^2 - 20$, where *h* represents the height above the cliff.

Chapter 4 Chapter Test Question 10 Page 205

a)



b) $h = -4.9t^2 + 5t + 2$

The *h*-intercept is 2. This represents the height, in metres, of the volleyball when it was hit.

c) From the graph, the volleyball hit the ground at 1.3 s. This is the *x*-intercept. It represents the horizontal distance of the ball when *h* = 0.

Chapter 4 Chapter Test Question 11 Page 205

a) $N = 5000 \times 2^t$ $\quad$ $N = 5000 \times 2^t$ $\quad$ $N = 5000 \times 2^t$ $\quad$ $N = 5000 \times 2^t$

$= 5000 \times 2^2$ $\quad$ $= 5000 \times 2^3$ $\quad$ $= 5000 \times 2^4$ $\quad$ $= 5000 \times 2^5$

$= 20\,000$ $\quad$ $= 40\,000$ $\quad$ $= 80\,000$ $\quad$ $= 160\,000$

The number of ants after 2, 3, 4, and 5 years is, respectively, 20 000, 40 000, 80 000, and 160 000.

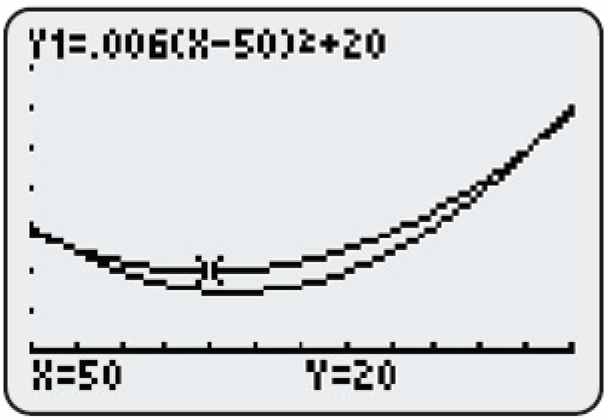
b) $t = 0$ represents July 1, the present time. $t = –2$ represents two years ago.

c)

$$\begin{aligned} N &= 5000 \times 2^t \\ 625 &= 5000 \times 2^t \\ \frac{625}{5000} &= 2^t \\ \frac{1}{8} &= 2^t \\ 2^{-3} &= 2^t \end{aligned}$$

$t = –3$. There were 625 ants 3 years ago.

Chapter 4 Chapter Test Question 12 Page 205



For speeds from 0 km/h to 17.1 km/h, the cost of operating the first car is less than that of the second car.

For speeds from 17.1 km/h to 122.9 km/h, the cost of operating the second car is less.

The first car is most efficient, at 20¢/km, driving at 50 km/h, and the second car is most efficient, at 15¢/km, driving at 55 km/h.

Chapter 4 Chapter Test Question 13 Page 205

Solutions for the Achievement Checks are shown in the Teacher's Resource.