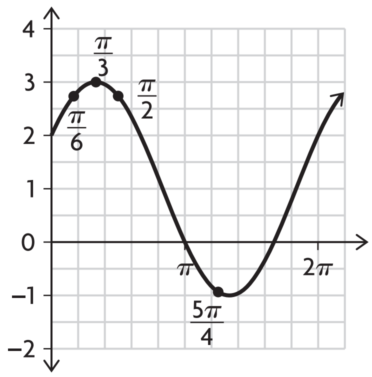
6.7 Rates of Change in Trigonometric Functions, pp. 369–373

1. a) The average rate of change is zero in the intervals of $0 < x < \pi$, $\pi < x < 2\pi$.
b) The average rate of change is negative in the intervals of $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $\frac{3\pi}{2} < x < \frac{5\pi}{2}$.
c) The average rate of change is positive in the intervals of $\frac{\pi}{2} < x < \frac{3\pi}{2}$, $\frac{5\pi}{2} < x < 3\pi$.
2. a) Two points where the instantaneous rate of change is zero are $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$.
b) Two points where the instantaneous rate of change is a negative value are $x = \frac{\pi}{2}$, $x = \frac{5\pi}{2}$.
c) Two points where the instantaneous rate of change is a positive value are $x = 0$, $x = 2\pi$.
3. Average rate of change for the interval $2 \le x \le 5$:

$$\left|\frac{0-0}{5-2}\right| = \left|\frac{0}{3}\right| = 0$$

4.



a) $0 \le x \le \frac{\pi}{2}$

$$\begin{aligned}\text{Average rate of change} &= \frac{2.73 - 2}{\frac{\pi}{2} - 0}\\ &= \frac{0.73}{1.57}\\ &\doteq 0.465\end{aligned}$$

b) $\frac{\pi}{6} \le x \le \frac{\pi}{2}$

$$\begin{aligned}\text{Average rate of change} &= \frac{2.73 - 2.73}{\frac{\pi}{2} - \frac{\pi}{6}}\\ &= \frac{0}{1.047}\\ &= 0\end{aligned}$$

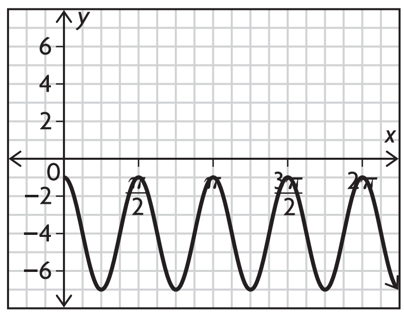
c) $\frac{\pi}{3} \le x \le \frac{\pi}{2}$

$$\begin{aligned}\text{Average rate of change} &= \frac{2.73 - 3}{\frac{\pi}{2} - \frac{\pi}{3}}\\ &= \frac{-0.27}{0.5236}\\ &\doteq -0.5157\end{aligned}$$

d) $\frac{\pi}{2} \le x \le \frac{5\pi}{4}$

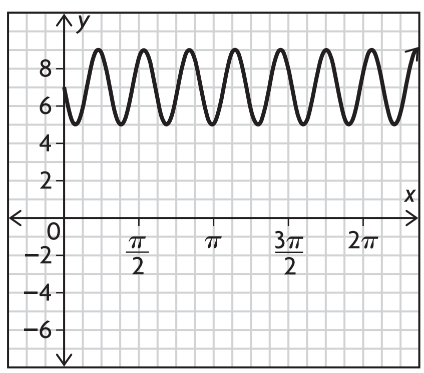
$$\begin{aligned}\text{Average rate of change} &= \frac{-0.932 - 2.73}{\frac{5\pi}{4} - \frac{\pi}{2}}\\ &= \frac{-3.662}{2.356}\\ &\doteq -1.554\end{aligned}$$

5.



a) The average rate of change is zero in the intervals of $0 < x < \frac{\pi}{2}$, $\pi < x < \frac{3\pi}{2}$.
b) The average rate of change is negative in the intervals of $0 < x < \frac{\pi}{4}$, $\pi < x < \frac{5\pi}{4}$.
c) The average rate of change is positive in the intervals of $\frac{\pi}{4} < x < \frac{\pi}{2}$, $\frac{5\pi}{4} < x < \frac{3\pi}{2}$.

6.



a) Two points where the instantaneous rate of change is zero are $x = \frac{1}{4}$, $x = \frac{3}{4}$.
b) Two points where the instantaneous rate of change is a negative value are $x = 0$, $x = 1$.
c) Two points where the instantaneous rate of change is a positive value are $x = \frac{1}{2}$, $x = \frac{3}{2}$.

7. a) $y = 6\cos(3x) + 2$ for $\frac{\pi}{4} \le x \le \pi$

For $x = \frac{\pi}{4}$,

$y = 6\cos\left(3\left(\frac{\pi}{4}\right)\right) + 2$

$y \doteq -2.2426$

For $x = \pi$,

$y = 6\cos(3(\pi)) + 2$

$y = -4$

$$\text{Average rate of change} = \frac{-2.2426 - (-4)}{\frac{\pi}{4} - \pi} = \frac{1.7574}{-2.3562} \doteq -0.7459$$

b) $y = -5\sin\left(\frac{1}{2}x\right) - 9$ for $\frac{\pi}{4} \le x \le \pi$

For $x = \frac{\pi}{4}$,

$y = -5\sin\left(\frac{1}{2}\left(\frac{\pi}{4}\right)\right) - 9$

$y \doteq -10.9134$

For $x = \pi$,

$y = -5\sin\left(\frac{1}{2}\pi\right) - 9$

$y = -14$

$$\text{Average rate of change} = \frac{-10.9134 - (-14)}{\frac{\pi}{4} - \pi} = \frac{3.0866}{-2.3562} \doteq -1.310$$

c) $y = \frac{1}{4}\cos\ (8x) + 6$ for $\frac{\pi}{4} \le x \le \pi$

For $x = \frac{\pi}{4}$,

$y = \frac{1}{4}\cos\left(8\left(\frac{\pi}{4}\right)\right) + 6$

$y = 6.25$

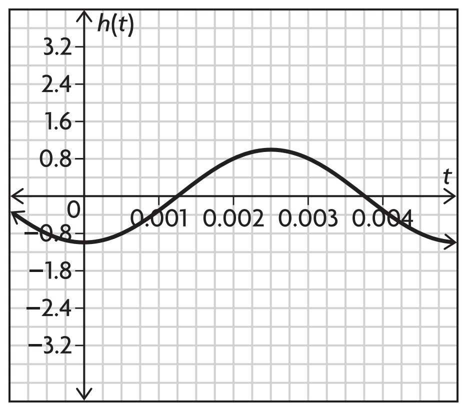
For $x = \pi$,

$y = \frac{1}{4}\cos\ (8\pi) + 6$

$y = 6.25$

$$\text{Average rate of change} = \frac{6.25 - 6.25}{\frac{\pi}{4} - \pi} = \frac{0}{-2.3562} = 0$$

8. The tip is at its minimum height at $t = 0$. Normally the sine function is 0 at 0, so the function in this case is translated to the right by $\frac{1}{4}$ of its period. The propeller makes 200 revolutions per second, so the period is $\frac{1}{200}$. The amplitude of the function is the length of the propeller, which is positive. Assume that it is 1 m for this exercise. Then the function that describes the height of the tip of the propeller is $h(t) = \sin(400\pi(t - \frac{1}{800}))$. The graph of this function is shown below.



$t = \frac{1}{300} \doteq 0.0033$

From the graph, it is clear that the instantaneous rate of change at $t = \frac{1}{300}$ is negative.

9. a) The axis is at 20.2. So, the equation of the axis is $y = 20.2$ and the amplitude is 4.5.

$k = \frac{2\pi}{24} = \frac{\pi}{12}$

$R(t) = 4.5\cos\left(\frac{\pi}{12}t\right) + 20.2$

b) fastest: $t = 6$ months, $t = 18$ months, $t = 30$ months, $t = 42$ months;

slowest: $t = 0$ months, $t = 12$ months, $t = 24$ months, $t = 36$ months, $t = 48$ months

c) For $5 \le t \le 7$

$t = 5$

$R(5) = 4.5\cos\left(\frac{\pi}{12}(5)\right) + 20.2$

$R(5) \doteq 21.364$

$t = 7$

$R(7) = 4.5 \cos\left(\frac{\pi}{12}(7)\right) + 20.2$

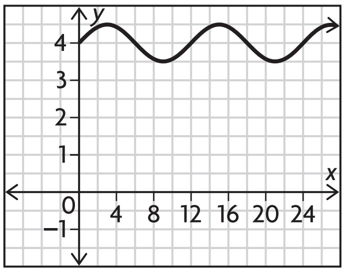
$R(7) \doteq 19.035$

Use the points (5, 21.364) and (7, 19.035).

$\frac{19.035 - 21.364}{7 - 5} \doteq \frac{-2.329}{2} \doteq -1.164$

$\doteq 1.164$ mice per owl/s

10. a)



The instantaneous rate of change appears to be at its greatest at 12 hours.

i) For $11 \leq t \leq 13$

$t = 11$

$y = 0.5 \sin\left(\frac{\pi}{6}(11)\right) + 4$

$y = 3.75$

$t = 13$

$y = 0.5 \sin\left(\frac{\pi}{6}(13)\right) + 4$

$y = 4.25$

Use the points (11, 3.75) and (13, 4.25).

$\frac{4.25 - 3.75}{13 - 11} = \frac{0.5}{2} = 0.25$ t/h

ii) For $11.5 \leq t \leq 12.5$

$t = 11.5$

$y = 0.5 \sin\left(\frac{\pi}{6}(11.5)\right) + 4$

$y \doteq 3.8706$

$t = 12.5$

$y = 0.5 \sin\left(\frac{\pi}{6}(12.5)\right) + 4$

$y \doteq 4.1294$

Use the points (11.5, 3.8706) and (12.5, 4.1294).

$\frac{4.1294 - 3.8706}{12.5 - 11.5} \doteq 0.2588$ t/h

iii) For $11.75 \leq t \leq 12.25$

$t = 11.75$

$y = 0.5 \sin\left(\frac{\pi}{6}(11.75)\right) + 4$

$y \doteq 3.9347$

$t = 12.25$

$y = 0.5 \sin\left(\frac{\pi}{6}(12.25)\right) + 4$

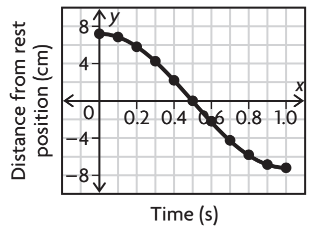
$y \doteq 4.0653$

Use the points (11.75, 3.9347 and (12.25, 4.0653).

$\frac{4.0653 - 3.9347}{12.25 - 11.75} = \frac{0.1306}{0.5} = 0.2612$ t/h

b) The estimate calculated in part iii) is the most accurate. The smaller the interval, the more accurate the estimate.

11. a)



b) half of one cycle

c) $\frac{-7.2 - 7.2}{1 - 0} = -14.4$ cm/s

d) The bob is moving the fastest when it passes through its rest position. You can tell because the images of the balls are farthest apart at this point.

e) The pendulum's rest position is halfway between the maximum and minimum values on the graph. Therefore, at this point, the pendulum's instantaneous rate of change is at its maximum.

12. $h(t) = \sin\left(\frac{\pi}{5}t\right)$

a) For $0 \leq t \leq 5$,

$h(0) = \sin\left(\frac{\pi}{5}(0)\right) = 0$

$h(5) = \sin\left(\frac{\pi}{5}(5)\right) = 0$

$\frac{0 - 0}{5 - 0} = 0$

b) For $5.5 \leq t \leq 6.5$

$t = 5.5$

$h(5.5) = \sin\left(\frac{\pi}{5}(5.5)\right)$

$h(5.5) = -0.309$

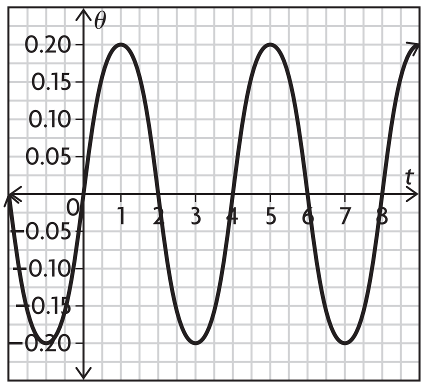
$h(6.5) = \sin\left(\frac{\pi}{5}(6.5)\right)$

$h(6.5) \doteq -0.809$

Use the points $(5.5, -0.309)$ and $(6.5, -0.809)$.

$$\frac{-0.809 - (-0.309)}{6.5 - 5.5} = \frac{-0.5}{1} = -0.5 \text{ m/s}$$

13. a)



b) When $t = 0, \theta = 0$. When $t = 1,\ \theta = 0.2$. The average rate of change is 0.2 radians/s.

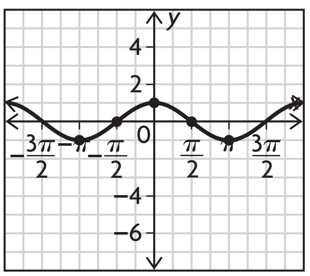
c) Answers may vary. For example, from the graph, it appears that the instantaneous rate of at $t = 1.5$ is about $-\frac{2}{3}$ radians/s.

d) The pendulum speed seems to be the greatest for $t = 0, 2, 4, 6$, and 8.

14. Answers may vary. For example, for $x = 0$, the instantaneous rate of change of $f(x) = \sin x$ is approximately 0.9003, while the instantaneous rate of change of $f(x) = 3 \sin x$ is approximately 2.7009. (The interval $-\frac{\pi}{4} < x < \frac{\pi}{4}$ was used.) Therefore, the instantaneous rate of change of $f(x) = 3 \sin x$ is at its maximum three times more than the instantaneous rate of change of $f(x) = \sin x$. However, there are points where the instantaneous rate of change is the same for the two functions. For example, at $x = \frac{\pi}{2}$, it is 0 for both functions.

15. a) By examining the graph of $f(x) = \sin x$, it appears that the instantaneous rate of change at the given values of x are $-1, 0, 1, 0$, and -1.

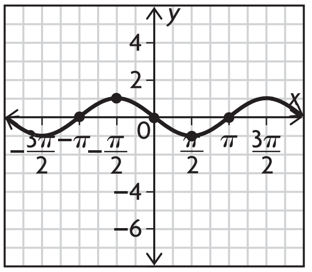
b)



The function is $f(x) = \cos x$. Based on this information, the derivative of $f(x) = \sin x$ is $\cos x$.

16. a) By examining the graph of $f(x) = \cos x$, it appears that the instantaneous rate of change at the given values of x are $0, 1, 0, -1$, and 0.

b)



The function is $f(x) = -\sin x$. Based on this information, the derivative of $f(x) = \cos x$ is $-\sin x$.

Chapter Review, pp. 376–377

1. Circumference $= 2\pi r = 2\pi(16) = 32\pi$

$$\frac{33}{32\pi} = \frac{x}{2\pi}$$
$$(32\pi)(x) = (33)(2\pi)$$
$$(32\pi)(x) = 66\pi$$
$$x = \frac{66\pi}{32\pi}$$
$$x = \frac{33}{16}$$

2. Circumference $= 2\pi r = 2\pi(75) = 150\pi$

$$\frac{x}{150\pi} = \frac{\frac{14\pi}{15}}{2\pi}$$
$$(2\pi)(x) = (150\pi)\left(\frac{14\pi}{15}\right)$$
$$(2\pi)(x) = 140\pi^2$$
$$x = 70\pi$$

3. a) $20° = 20° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{9}$ radians

b) $-50° = -50° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{-5\pi}{18}$ radians

c) $160° = 160° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{8\pi}{9}$ radians

d) $420° = 420° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{7\pi}{3}$ radians

4. a) $\frac{\pi}{4}$ radians;

$$\frac{\cancel{\pi}}{4} \cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = 45°$$